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S1. EXPANDED GOVERNING EQUATIONS

Conservation of mass, water, and enthalpy within the magma chamber lead to the following three equations governing chamber pressure P_c, temperature T, and the volume fraction of exsolve volatiles ε_g:

Conservation of mass:

$$(\beta+\beta_r)\frac{dP_c}{dt}+\left(-\alpha-\alpha_r+\frac{\rho_x-\rho_m}{\rho}\frac{\partial\varepsilon_x}{\partial T}\right)\frac{dT}{dt}+\left(\frac{\rho_g-\rho_m}{\rho}+\frac{\rho_x-\rho_m}{\rho}\frac{\partial\varepsilon_x}{\partial\varepsilon_g}\right)\frac{d\varepsilon_g}{dt}=-\frac{q_{dike}}{\rho V_c}-\frac{\Delta P_c}{\eta_r} \quad (1)$$

Conservation of water:

$$\left(\frac{1}{\rho_g}\frac{d\rho_g}{dP}+\beta_r+\frac{m_{eq}\rho_m\varepsilon_m}{\rho_g\varepsilon_g}\times\left[\frac{1}{m_{eq}}\frac{\partial m_{eq}}{\partial P}+\beta_m+\beta_r\right]\right)\frac{dP_c}{dt}+\left(\frac{1}{\rho_g}\frac{d\rho_g}{dT}-\alpha_r+\frac{m_{eq}\rho_m\varepsilon_m}{\rho_g\varepsilon_g}\times\left[\frac{1}{m_{eq}}\frac{\partial m_{eq}}{\partial T}-\alpha_m-\alpha_r-\frac{1}{\varepsilon_m}\frac{\partial\varepsilon_x}{\partial T}\right]\right)\frac{dT}{dt}+\left(\frac{1}{\varepsilon_g}-\frac{m_{eq}\rho_m}{\rho_g\varepsilon_g}\left[1+\frac{\partial\varepsilon_x}{\partial\varepsilon_g}\right]\right)\frac{d\varepsilon_g}{dt}=\frac{q_{dike}}{\rho_g\varepsilon_g V_c}\left(\frac{m_{eq}\rho_m\varepsilon_m}{\rho}+\frac{\rho_g\varepsilon_g}{\rho}\right)-\frac{\Delta P_c}{\eta_r}\left(1+\frac{m_{eq}\rho_m\varepsilon_m}{\rho_g\varepsilon_g}\right) \quad (2)$$

Conservation of enthalpy:

$$\left(\frac{P_c\beta_r}{\rho cT}+\beta+\frac{1}{c}\frac{\partial c}{\partial P}+\beta_r-\frac{L_m\rho_x\varepsilon_x}{\rho cT}[\beta_x+\beta_r]-\frac{L_e m_{eq}\rho_m\varepsilon_m}{\rho cT}\left[\frac{1}{m_{eq}}\frac{\partial m_{eq}}{\partial P}+\beta_m+\beta_r\right]\right)\frac{dP_c}{dt}+$$

$$\left(-\frac{\alpha_r P_c}{\rho cT}-\alpha+\frac{\rho_x-\rho_m}{\rho}\frac{\partial\varepsilon_x}{\partial T}+\frac{1}{c}\frac{\partial c}{\partial T}+\frac{1}{T}-\alpha_r-\frac{L_m\rho_x\varepsilon_x}{\rho cT}\left[-\alpha_x+\frac{1}{\varepsilon_x}\frac{\partial\varepsilon_x}{\partial T}-\alpha_r\right]-\frac{L_e m_{eq}\rho_m\varepsilon_m}{\rho cT}\left[\frac{1}{m_{eq}}\frac{\partial m_{eq}}{\partial T}-\right.$$

$$\alpha_m - \frac{1}{\varepsilon_x}\frac{\partial \varepsilon_x}{\partial T} - \alpha_r\bigg]\bigg)\frac{dT}{dt} + \left(\frac{\rho_g - \rho_m}{\rho} + \frac{\rho_x - \rho_m}{\rho}\frac{\partial \varepsilon_x}{\partial \varepsilon_g} + \frac{1}{c}\frac{\partial c}{\partial \varepsilon_g} - \frac{L_m \rho_x}{\rho c T}\frac{\partial \varepsilon_x}{\partial \varepsilon_g} + \frac{L_e m_{eq} \rho_m}{\rho c T}\left[1 + \frac{\partial \varepsilon_x}{\partial \varepsilon_g}\right]\right)\frac{d\varepsilon_g}{dt} =$$

$$-\frac{\dot{H}_{out}}{\rho c T V_c} - \frac{\Delta P_c}{\eta_r}\left(1 - \frac{L_m \rho_x \varepsilon_x}{\rho c T} - \frac{L_e m_{eq} \rho_m \varepsilon_m}{\rho c T} - \frac{P_c}{\rho c T}\right) \quad (3)$$

Conservation of mass in the dike leads to an equation for dike overpressure ΔP_d (Segall et al., 2001):

$$\frac{d\Delta P_d}{dt} = \frac{3\mu c E(k)}{2\pi(1-\nu)\rho_m}\frac{(P_c - P_d)}{ab^2} - (1 - A)\frac{\Delta P_d}{a}\frac{da}{dt} - (2 + A)\frac{\Delta P_d}{b}\frac{db}{dt} \quad (4)$$

$$A = \left(\frac{E-F}{E}\right)\left(\frac{a^2}{b^2} - 1\right)^{-1} \quad (5)$$

F is the complete elliptic integral of the first kind with modulus k.

If the dike breaches the surface ($b = d$), we must account for the eruptive flux q_{erupt}, which will depend on the vertical pressure gradient, the dike aperture, and the length of the eruptive fissure, given as some fraction of the total dike length ω with $0 < \omega \leq 2a$ (Segall et al., 2001):

$$q_{erupt} = \frac{2(1-\nu)^3 d^2 \omega a {\Delta P_d}^3}{3\eta\mu^3 E^3(k)}(\Delta P_d - \Delta\rho g d) \quad (6)$$

$$\Delta P_d > \Delta\rho g d$$

In this case, the pressure evolution in the dike is given by:

$$\frac{d\Delta P_d}{dt} = \frac{3\mu E(k)}{2\pi(1-\nu)ad^2}\left[\frac{c}{\rho_m}(P_c - P_d) - \frac{\delta^3 a}{48\eta d}(\Delta P_d - \Delta\rho g d)\right] - (1 - A)\frac{\Delta P_d}{a}\frac{da}{dt} \quad (7)$$

A list of symbol meanings and values used is provided in Table S1.

Table S1

Symbol	Definition	Value or initial condition
a	Dike length	$a_0 = 0.02d$
b	Dike height	$b_0 = a_0/2$
c_m, c_x, c_g	Specific heat capacity of melt, crystals, and volatiles	$c_m = c_x = 1200$ J kg^{-1} K^{-1} $c_g = 3880$ J kg^{-1} K^{-1}
c	Mixture specific heat capacity	$\frac{1}{\rho}\left(c_m\rho_m\varepsilon_m + c_x\rho_x\varepsilon_x + c_g\rho_g\varepsilon_g\right)$
d	Depth to chamber	4 – 10 km
g	Gravitational acceleration	9.81 m/s^2
L_e, L_m	Latent heat of exsolution and melting	$L_e = 610 \times 10^3$ J kg^{-1} $L_m = 290 \times 10^3$ J kg^{-1}
m_{eq}	Solubility of water in melt	parameterization of Dufek and Bergantz (2005) and Zhang (1999)
P_c, P_d	Pressure in the chamber and dike	$P_{c,0} = P_{d,0} = P_{lit} + P_{crit}$
P_{crit}	Critical overpressure to initiate dike	10 – 50 MPa
P_{lit}	Lithostatic pressure	$P_{lit} = \rho_r g d$
T	Temperature of the magma	$T_0 = 1200$ K
V_{ch}	Volume of the chamber	$V_{c,0} = 10^{-2} – 10^{2}$ km^3
$\alpha_x, \alpha_m, \alpha_r$	Thermal expansion of crystals, melt, and crust	10^{-5} K^{-1}
α	Bulk thermal expansion of magma	$\frac{1}{\rho}\left(\alpha_m\rho_m\varepsilon_m + \alpha_x\rho_x\varepsilon_x - \varepsilon_g\frac{\partial\rho_g}{\partial T}\right)$
$\beta_x, \beta_m, \beta_r$	Compressibility of crystals, melt, and crust	10^{-10} Pa^{-1}
β	Effective compressibility of magma chamber	$\beta_r + \frac{1}{\rho}\frac{\partial\rho}{\partial P}$
γ	Dike-chamber conductivity	1 kg/Pa·s
δ	Dike aperture	$\frac{2b(1-\nu)\Delta P_d}{\mu E(k)}$
η	Magma viscosity	10^3 Pa·s

μ	Shear modulus of crust	10 GPa
ν	Poisson's ratio of crust	0.25
ρ_x, ρ_m, ρ_g	Density of crystals, melt, and exsolved volatiles	$\rho_{x,0} = 2600$ kg/m^3 $\rho_{m,0} = 2400$ kg/m^3 ρ_g calculated from Redlich-Kwong equation of state (Halbach and Chatterjee, 1982; Huber et al., 2010)
ρ_r	Density of crust	2750 kg/m^3
ρ	Bulk density of magma	$\rho_m \varepsilon_m + \rho_x \varepsilon_x + \rho_g \varepsilon_g$
ω	Fissure length	$0.5a$

S2. EFFECT OF MAGMA VISCOSITY

In Figure S1 we show example model calculations where we vary magma viscosity between 10^2 Pa·s – 10^7 Pa·s, using values of V_{ch} = 10 km^3, d = 8 km, H_2O = 6 wt%, and P_{crit} = 20 MPa. In the example, we show the time series for erupted volume, dike height and length, chamber and dike pressures, and volume fractions of crystals and exsolved volatiles. In all cases, the magma viscosity affects only the timescale over which the parameters evolve and does not impact their final values.

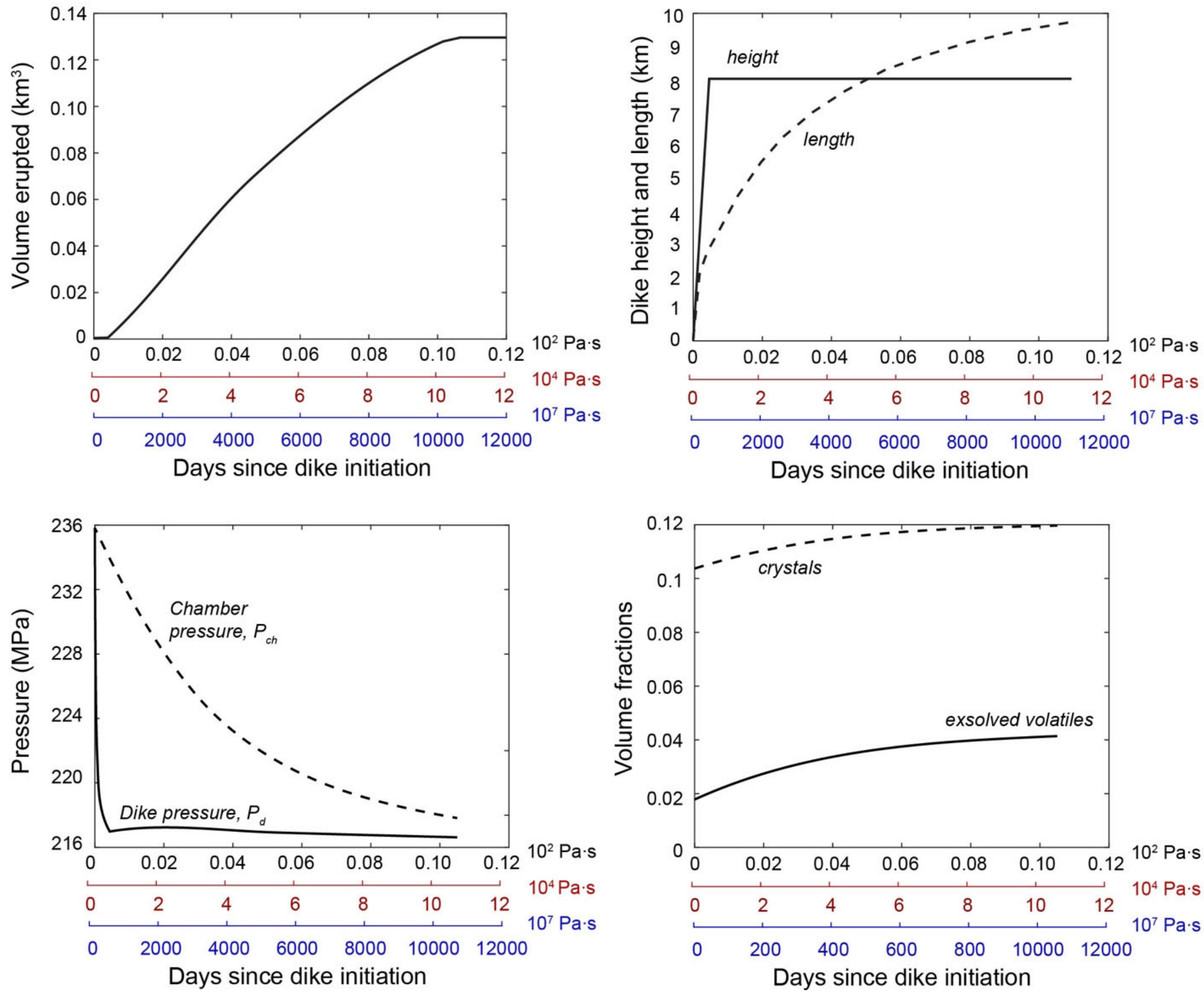

Figure S1. Model output of time series for erupted volume (top left), dike height and length (top right), dike and chamber pressure (bottom left), and the volume fractions of crystals and exsolved water (bottom right). Three different x-axes correspond to three different values of magma viscosity: 10^2 Pa·s (black), 10^4 Pa·s (red), and 10^7 Pa·s (blue).

S2. EFFECT OF DIKE-CHAMBER CONDUCTIVITY γ AND FISSURE LENGTH ω

The dike-chamber conductivity γ in theory depends on the size and shape of the pathway connecting the dike and chamber. In most cases the structure of this connection is unknown; in rare cases such as Kilauea Volcano, Hawaii, there is geophysical evidence that dikes are fed by a cylindrical-shaped conduit connected to the chamber (Segall et al., 2001). For a cylindrical conduit of radius r and length L, $\gamma = \pi\rho r^4/8\eta L$.

In Figure S2 (left), we show how the erupted volume V_{er} is affected by the choice of γ. The results are only impacted within the "dike-limited regime," where greater conductivity in general leads to greater dike volumes and hence smaller eruption volumes. This arises because the greater conductivity allows for slightly greater pressures in the dike, leading to slightly greater dike widths and hence dike volumes. As a result of greater intrusive volumes, the critical chamber volume for an eruption is slightly greater with greater γ. Outside of the dike-limited regime, the results for V_{er} are unaffected.

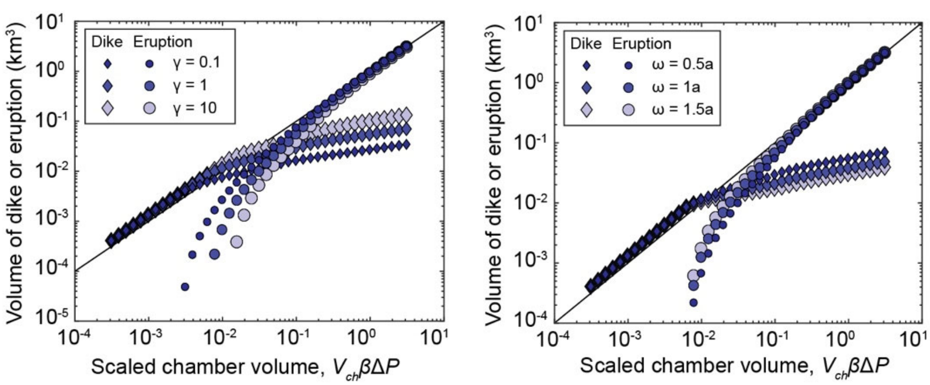

Figure S2. Plots of erupted or intruded volume versus

In Figure S2 (right), we show results for V_{er} for different values of the eruptive fissure length ω, defined as some fraction of the total dike length a. Changing ω from $0.5a$ to $1.5a$ results in a difference in erupted and dike volumes by at most a factor of 2-3.

S3. DEFINITION OF EFFECTIVE MAGMA COMPRESSIBLITY

The effective compressibility of the magma chamber, including the compressibility of the cavity and the magma is defined as:

$$\beta = \beta_r + \frac{1}{\rho}\frac{\partial \rho}{\partial P}$$

In terms of mass fractions n, the bulk magma density is

$$\rho = \left[\frac{1 - n_x - n_g}{\rho_m} + \frac{n_x}{\rho_x} + \frac{n_g}{\rho_g}\right]^{-1}$$

Since we assume $\beta_x = \beta_m$ and since $\rho_x \approx \rho_m$, in the absence of any exsolved volatiles, $\frac{1}{\rho}\frac{\partial \rho}{\partial P} \approx \beta_x$, so $\beta \approx 2 \times 10^{-10}$ Pa^{-1}. In other words, when there are no exsolved volatiles, the bulk compressibility is determined by the mechanical properties of the melt and crystals. When $n_g > 0$, the derivative of the mixture density with respect to pressure depends not only on mechanical properties but also on the solubility of the volatile phase:

$$\frac{\partial \rho}{\partial P} \approx -\rho^2 \frac{\partial}{\partial P}\left[\frac{1 - n_g}{\rho_x} + \frac{n_g}{\rho_g}\right]$$

$$= -\rho^2 \left[-\beta_x \frac{1 - n_g}{\rho_x} + \left(\frac{1}{\rho_x} - \frac{1}{\rho_g}\right)\frac{\partial m_{eq}}{\partial P} - \frac{n_g}{\rho_g}\beta_g\right]$$

An equation for $\frac{\partial m_{eq}}{\partial P}$ comes from Degruyter and Huber (2014, A.3), who parameterize water solubility in silicate melt following Dufek and Bergantz (2005) and Zhang (1999). The Redlich-Kwong equation of state for the gas phase is provided in Degruyter and Huber (2014, A.1) and can be used to estimate β_g. Over the range of conditions tested in this work, $\frac{\partial m_{eq}}{\partial P} \approx 2 \times 10^{-10}$ Pa^{-1}, $\rho_g \approx 400$ kg/m^3, and $\beta_g \approx 3 \times 10^{-9}$ Pa^{-1}. Thus, for an exsolved volatile mass fraction of about $n_g = 0.002$ (or a gas volume fraction of about 1%), the effective compressibility $\beta \approx 1 \times 10^{-9}$ Pa^{-1}.

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