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Supplementary Materials for

Linking postglacial landscapes to glacier dynamics using swath radar at Thwaites Glacier

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S1. HYDROLOGIC POTENTIAL AND WATER ROUTING

Preferred water pathways were computed assuming that water flows down hydraulic gradients, where hydraulic potential is set primarily by the glaciostatic pressure according to the following relation (Shreve, 1972):

$$\Phi = \rho_i g z_s + (\rho_i - \rho_i) g z_b \qquad \text{(Eq. 1)}$$

In this formulation, hydrologic potential (Φ) is dominated by the surface elevation profile (z_s) with secondary effects from bed elevation (z_b). Computation requires additional physical constants, including the densities of ice and water (ρ_i, ρ_w) and the acceleration due to gravity (g). Water routing (as presented in fig. 2a and fig. 2c) was calculated using a D-infinity routing scheme (Tarboton, 1997; Schwanghart and Scherler, 2014).

S2. MSGL WIDTH CONTROLLED BY THE VISCOUS FLOW OF TILL

Models of MSGL formation that rely entirely on viscous flow of till driven by subsurface pressure gradients are often heuristic in nature (Barchyn et al., 2016) – these models lack an intrinsic lateral transport mechanism and therefore must impose lateral transport rates. MSGL length in these models, however, depends critically on several ice-dynamic parameters: basal velocity, ice effective viscosity, MSGL height, and basal sediment entrainment/deposition rates. These relationships are primarily due to their control on the size of lee-side cavities, pressure lows behind bed features infilled by till flow to form MSGL. The relative rates of ice-till detachment, creep-closure, and entrained sediment deposition ultimately dictate MSGL length. *In these models, faster ice flow predicts longer features.*

S3. MSGL WIDTH CONTROLLED BY ICE FLOW USING A REINER-RIVLIN RHEOLOGY

Observations at the Thwaites Glacier bed show glacier bedforms that are generally aligned with the ice-flow direction, not the water-flow direction. Schoof and Clarke (2008) showed that, in the presence of simple shear at the base of an ice column, normal stresses are generated transverse to the primary flow direction. Assuming the ice and till are coupled, this drives sediment transport, with till divergence and convergence driving the formation of corrugations transverse to the flow direction.

Lateral transport rates in this model scale with the square of the basal shear stress. Without imposing a wavelength-dependent decay mechanism, narrow MSGL grow unstably – Schoof and Clarke argued that down-slope sediment transport would dampen the growth of short wavelength features, leading to a stable MSGL width subject to an unknown diffusive transport coefficient. *This model predicts that the equilibrium MSGL width should decrease with increasing basal shear stress*, as faster transverse flows driven by higher stresses begin to out-compete diffusion at higher wavenumbers.

The growth rate of those bedforms (σ) depends on a range of ice flow parameters (see Methods Table S1), but importantly it varies with basal shear stress (τ_b), basal velocity (u_b), and the wavenumber describing bedform spacing (k):

$$\sigma = \frac{d_s \tau_b^2 \mu k^2}{\eta^2 (2\eta |k| + \tau_b / u_b)} - \alpha k^2$$

(Eq. 2) [Source Eq. 60](Schoof and Clarke, 2008)

Table S1. - Parameter definitions for equation 2. (See Schoof and Clarke for the full derivation)	
σ	MSGL growth rate
d_s	Ploughing depth of the till
τ_b	Basal shear stress
μ	Reiner-Rivlin rheology scaler
k	MSGL transverse wavenumber
η	Ice viscosity
u_b	Basal velocity
α	Sediment transport coefficient (diffusivity)

S4. MSGL LENGTH AND WIDTH CONTROLLED BY THE HYDROLOGIC RILLING INSTABILITY

The final proposed instability mechanism invokes active drainage under ice sheets, with bed-load transport in subglacial streams thought to exhibit the same "rilling instability" as surface streams(Fowler, 2010; Fowler and Chapwanya, 2014). Greater channel depth results in higher shear-stress at the water sediment interface, moving more material downstream, driving a positive feedback that cuts channels along the water-flow direction. *Highly pressurized water systems (associated with low basal shear stress) will tend to increase both the width and length of MSGL*.

Effective pressure at the base of the ice sheet dictates the expected water-film thickness, which controls the stress balance at the water-till interface and the ultimate MSGL morphology under the rilling instability hypothesis. Following Fowler (2010), we compute the range of MSGL widths and lengths expected under West Antarctic ice streams. Increasing effective stress increases contact between ice and till, reduces water film thickness, and ultimately results in smaller MSGL

To initiate the rilling instability, effective shear-stresses imposed by water flow on till grains must exceed the critical Shields stress of the till. For reasonable grain sizes (>= 10^{-4} m), using the ice thickness, basal shear stress, and flow speed at Thwaites Glacier, that threshold is not met. Using smaller than predicted grain sizes ~O(10^{-5} m) and assuming no cohesion (which would be expected for, e.g., clay), the rilling instability is possible within our Thwaites domain. No prescribed till grain size results in unstable MSGL formation for the full range of basal shear stresses observed at upper and lower Thwaites Glacier. Ultimately, the optimum bedform geometries are explained by Equations 3 and 4.

$$MSGL_w = \pi l\sqrt{2}\left[\frac{M}{2}\frac{\alpha\kappa}{\sigma}\right]^{1/3}\left(\frac{\tau_c^*}{\upsilon}\right)^{1/2}$$ (Eq. 3) [Source Eq. 4.7](Fowler, 2010)

$$MSGL_l = \frac{3\pi l\kappa\sigma^{1/3}\tau^{*3/2}\sqrt{L}}{2\sqrt{2}\delta}$$ (Eq. 4) [Source Eq. 4.8](Fowler, 2010)

Table S2. - Parameter definitions for equations 3,4. (See Fowler for the full derivation)

$MSGL_w$	Optimum MSGL width
$MSGL_l$	Optimum MSGL length
l	Bedform length scale (function of velocity, viscosity, and water pressure)
M	2, defined by the limiting case where bedforms are long relative to ice thickness
α	Ratio of bedform height and deforming till depth (function of water pressure)
κ	Dimensionless scaler
σ	Dimensionless scaler (bedform length scale / ice thickness)
τ_c^*	Critical Shields stress
υ	Aspect ratio of corrugation (function of water pressure)
τ^*	Normalized effective stress (function of water pressure)
L	0.1, Nondimensional scaling factor for MSGL length
δ	Ratio of water film thickness to bedform elevation

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