

## DATA REPOSITORY

The governing equations for HRAM data processing and forward modeling are listed below in the order of their appearance in the main text.

### REDUCTION TO POLE

The data are reduced to pole using a filter in the Fourier domain. This filter migrates the observed field from the observed magnetic inclination and declination, to what the field would look like at the magnetic pole. The transfer function of reduction to the magnetic pole can be expressed in the form of

$$L(f) = \frac{1}{[Nn - Mm + j \operatorname{sgn}(f)(Nm + Mn)]} \quad (1)$$

where  $f$ , is the spatial frequency,  $\operatorname{sgn}(f)$  is the sign of  $f$ ,  $j$  is the imaginary unit.  $M$ ,  $N$ ,  $m$  and  $n$  are the direction cosines of the magnetization and of the earth's magnetic field, respectively (Blakely and Cox, 1972). The direction cosines can be expressed by the inclination  $a$  and declination  $b$  of the magnetization, and by the inclination  $I$  and declination  $D$  of the earth's magnetic field

$$\begin{aligned} M &= \cos a \cos(A - b), N = \sin a, \\ m &= \cos a \cos(A - D), N = \sin I \end{aligned} \quad (2)$$

where  $A$  is the azimuth of the profile measured clockwise from geographic north.

### BUTTERWORTH FILTER

If  $k$ ,  $k_0$  and  $n$  is the wavenumber, central wavenumber of the filter, and degree of the Butterworth filter function, respectively, the filter is simply defined as

$$L(k) = \frac{1}{\left[1 + \left(\frac{k}{k_0}\right)^n\right]} \quad (3)$$

## POWER SPECTRUM ANALYSIS

The Spector and Grant (1970) equation is

$$|F(k)|^2 = 4\pi^2 C_m^2 |\theta_m|^2 |\theta_f|^2 M_0^2 e^{-2|k|z_t} (1 - e^{-|k|(z_b - z_t)})^2 S^2(a, b) \quad (4)$$

where  $F$  is the Fourier power spectrum,  $k$  is wavenumber in cycles  $\text{km}^{-1}$  or  $2\pi \text{ km}^{-1}$ ,  $C_m$  is a constant related to units,  $\theta_m$  is a factor related to magnetization direction,  $\theta_f$  is a factor related to magnetic field direction,  $M_0$  is magnetization,  $Z_t$  and  $Z_b$  are the depths to the top and the bottom of the ensemble of magnetic sources, and  $S^2(a, b)$  is the factor related to horizontal dimensions of sources.

## HILBERT TRANSFORM

Hilbert transform is defined as a composition of two part acting on the x component and one part on the y component and therefore the magnetic vertical and horizontal derivatives as the Hilbert transforms of each other (Nabighian, 1984). The generalized relationship can be presented as

$$H(M) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} M(\xi, \eta) \nabla_h \left( \frac{1}{R} \right) d\xi d\eta \quad (5)$$

where  $(\xi, \eta)$  is the integration point in the x, y plane,  $\nabla_h$  is the horizontal gradient at the integration point, and  $R$  is the integration point residual.

## VERTICAL AND TILT DERIVATIVES

The first order vertical derivative (VDRV) of the magnetic field TMAG-RTP is calculated using the  $z$  component of the wavenumber and can be shown as follows

$$VDRV = \left( \frac{\partial T}{\partial z} \right) \quad (6)$$

The tilt derivative (TDRV) or tilt angle is the ratio of the vertical and total horizontal (THDRV) derivatives (Miller and Singh, 1994; Verduzco et al., 2004), given by

$$TDRV = \tan^{-1} \left( \frac{VDRV}{THDRV} \right) \quad (7)$$

where the THDRV is computed using the first derivatives of the magnetic field in the x and y directions, given by

$$THDRV = \sqrt{\left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2} \quad (8)$$

## VERTICAL FAULT MODELING

The vertical fault is simulated by two semi-infinite horizontal 2D sheets with 50 m offset. The vertical fault is modeled by (Telford et al., 1990)

$$\Delta T(x) = -2KtT_0 \left[ \left( \frac{1}{r_1^2} \right) \{Z_s \sin 2l \sin \beta - (X + Z_s \cos \varepsilon)(\cos^2 l \sin^2 \beta - \sin^2 l)\} \right. \\ \left. - \left( \frac{1}{r_2^2} \right) \{Z_d \sin 2l \sin \beta - (X + Z_d \cos \varepsilon)(\cos^2 l \sin^2 \beta - \sin^2 l)\} \right] \quad (9)$$

where  $K$ ,  $t$ , and  $T_0$  are magnetic susceptibility contrast, thickness of the sheets, and total geomagnetic intensity in the area, respectively.  $Z_d$  and  $Z_s$  represent the depth to the deeper and shallower sheets. The  $\varepsilon$  and  $\beta$  are the angles related to the dip and strike of the fault plane. The  $r_1$  and  $r_2$  are the distance from the tip of the sheets at the fault plane to the surface where the magnetic anomaly ( $P$ ) is calculated.

## DIKE MODELING

The dike signature is modeled by a 2D vertical prism using the general equations by Hood (1964) and Ram Babu et al. (1986)

$$\Delta T(x) = C \left[ \cos \theta \left\{ \tan^{-1} \left( \frac{X+B}{H} \right) - \tan^{-1} \left( \frac{X-B}{H} \right) \right\} \right. \\ \left. + \frac{1}{2} \sin \theta \log_e \frac{(X+B)^2 + H^2}{(X-B)^2 + H^2} \right] \quad (10)$$

where  $C$  and  $\theta$  are the amplitude coefficient and the index parameter, respectively.  $X$ ,  $B$ , and  $H$  are the distance of the point of observation from the origin, half-thickness, and depth to the top of the dike from the plane of observation.

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