Vegetation control allows autocyclic formation of multiple dunes on prograding coasts

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1 Model of vegetated dunes on prograding coast

Here we describe a modified version of the Coastal Dune Model (Duran and Moore, 2013). Modifications are highlighted as *new*. The model describes the temporal evolution of the sand surface elevation $h(x, y, t)$– defined relative to the mean high water level (MHWL)–and the cover fraction $\rho_{\mathrm{veg}}(x, y, t)$ for a single generic grass species. x is the cross-shore distance to the shoreline ($x = 0$), which separates the foreshore ($x < 0$) from the backshore ($x > 0$), and y is the alongshore coordinate.

1.1 Fluid dynamics

The model uses a linear solution of the Reynolds averaged Navier-Stokes equations for the turbulent boundary layer over smooth terrain to calculate the perturbation $\delta\boldsymbol{\tau}$ of the undisturbed surface wind shear stress $\boldsymbol{\tau}_0$ induced by the topography h. The surface shear stress $\boldsymbol{\tau}$ is $\boldsymbol{\tau}(h) = \boldsymbol{\tau}_0 + \tau_0\delta\boldsymbol{\tau}(h)$ (see Ref. 1 for details). For lee slopes steeper than the separation angle $\sim 20°$, non-linear hydrodynamic effects are simply modeled by a separation streamline below which wind and flux are set to zero. Each streamline is defined by a third-order polynomial connecting the brink with the ground at the reattachment point [2].

1.2 Shear stress partition

In the presence of vegetation, plants act as roughness elements that absorb part of the momentum transferred to the sand surface by the wind, effectively reducing the surface shear stress and thus the sand transport rate. For randomly distributed plants, and assuming the effective shelter area for one plant is proportional to its basal area, the fraction τ_s of the surface shear stress acting on the sand decreases with the local vegetation

cover fraction ρ_{veg} as

$$\tau_s = \tau/(1 + \Gamma \rho_{\text{veg}}) \tag{1}$$

where Γ is a dimensionless 'roughness factor' that describes the effectiveness of the vegetation in slowing down the flow and thus in trapping sand. In the model, $\Gamma = 16$ is calculated from values of plant form drag and geometry reported for creosote communities (see Ref. 1 and references therein; it is reasonable to expect a similar value for coastal grasses and desert bushes due to a roughly similar plant geometry).

1.3 Effect of wetting on the transport threshold

We further consider that at the shore, transport is naturally limited by the elevation h relative to the watertable, as the transport threshold τ_t is much higher for wet grains than for dry ones. This relation is captured by the simple phenomenological expression

$$\tau_t(h) = \tau_t^d + (\tau_t^w - \tau_t^d) \exp\left(-(h + H_{\text{water}})/\delta_w\right) \tag{2}$$

where τ_t^d and $\tau_t^w = 10\tau_t^d$ are the thresholds for dry and wet sand respectively, H_{water} is the watertable depth relative to the MHWL and $\delta_w = 0.05$m characterizes the decrease in water content of the sand as a function of elevation. At the watertable ($h = -H_{\text{water}}$, relative to the MHWL) $\tau_t = \tau_t^w$ by definition, whereas far above it $\tau_t \to \tau_t^d$.

1.4 Aeolian sand transport

The sand flux is determined from the shear stress at the sand surface τ_s (Eq.1), the surface gradient ∇h and the transport threshold τ_t (Eq.2). It is well known that the sand flux $\mathbf{q}_a$ over an erodible surface increases with the distance downwind as the saltation process spatially adjusts to the wind forcing. This effect is modeled as

$$\nabla \cdot \mathbf{q}_a = (q_a + \delta_a)(1 - q_a/q_{\text{sat}})/l_{\text{sat}} \tag{3}$$

which describes the spatial relaxation of the sand flux toward an equilibrium 'saturated' value q_{sat} over a 'saturation' length l_{sat}. The saturated flux and saturation length are defined as: $q_{\text{sat}} = Q(\tau_s - \tau_t)/\tau_t^d$ and $l_{\text{sat}} = L\tau_t^d/(\tau_s - \tau_t)$, where $Q(\nabla h)$ and $L(\nabla h)$ are slope–dependent dimensional functions [2]. The small

term $\delta_a \sim 10^{-2} q_{\text{sat}}$ quantifies the direct sand entrainment in the absence of transport when the bed shear stress has just crossed the transport threshold. For simplicity in the formulation q_a is defined as a volume, not mass, flux.

1.5   Surface dynamics *(new)*

1.5.1   Foreshore ($x < 0$)

We assume the foreshore to be always at equilibrium ($\partial h / \partial t = 0$) with a constant shape defined by the initial condition. This assumption implies aeolian erosion is balanced by accretion in the swash zone. As a result, the simulated foreshore acts as a sand reservoir supplying an unlimited amount of sediment to the backshore, effectively feeding dune formation and post-storm recovery.

1.5.2   Shoreline ($x = 0$)

The shoreline migrates seaward (progrades) at an imposed rate $\dot{x}_{\text{shore}} > 0$.

1.5.3   Backshore ($x > 0$)

We calculate the change of the sand surface elevation h at the backshore from mass conservation as

$$\partial_t h = -\nabla \cdot \mathbf{q}_a - \dot{x}_{\text{shore}} \partial_x h. \tag{4}$$

The last term in the right hand side of Eq. 4 describes the effect of shoreline progradation.

1.5.4   Avalanches

For slopes steeper than the angle of repose $34°$, an additional gravity flow models the surface relaxation due to avalanches [2].

1.6   Vegetation dynamics *(new)*

Vegetation is characterized by the cover fraction ρ_{veg} at time t and position (x, y), where x is the distance to the shoreline and y the alongshore coordinate. Vegetation growth rate can be written as a balance between

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'vertical' vegetation growth rate G_v, lateral vegetation propagation rate G_l and mortality rate D:

$$\partial_t \rho_{\text{veg}} = G_v + G_l - D \tag{5}$$

where the different rates are function of the cover fraction ρ_{veg}, local erosion/deposition rate $\partial_t h$, slope $|\nabla h|$ and cover fraction gradient $\nabla \rho_{\text{veg}}$.

1.6.1 Vertical vegetation growth

We call 'vertical' growth any increase in cover fraction due to local biomass production. We assume this growth rate can be modeled in first approximation by a logistic equation, i.e., an initial exponential growth followed by a saturation at maximum cover $\rho_{\text{veg}} = 1$,

$$G_v = G_0 \rho_{\text{veg}}(1 - \rho_{\text{veg}}) \tag{6}$$

Following a typical response of dune–building plants to sand accretion, we assume the characteristic growth rate G_0 (units of inverse time) increases linearly with the local deposition rate $\partial_t h$ (for $\partial_t h > 0$). Furthermore, the growth rate is zero if the vegetation is too close to the shoreline, a limit characterized by the length L_{veg}. Thus,

$$G_0 = H_v^{-1} \, \partial_t h \; \Theta(\partial_t h) \; \Theta(x - L_{\text{veg}}) \tag{7}$$

where the Heaviside function $\Theta(x)$ equals 1 for $x > 0$, and 0 otherwise. The length H_v can be interpreted as the ratio of vegetation volume to the cover area (i.e. an effective vegetation height): smaller values imply the biomass production, stimulated by a given deposition rate, translates more efficiently into a higher cover area. In the model we set $H_v = 0.3$m and $L_{\text{veg}} = 20$m.

1.6.2 Mortality rate

We call 'mortality' any decrease in cover fraction induced by sand erosion $\partial_t h$ (for $\partial_t h < 0$). We assume in first approximation that overall mortality is dominated by erosion as it seems to be the case for most dune–building plants. Furthermore, we consider the mortality rate, just like the growth rate, scales with the

cover fraction ρ_{veg} as sand erosion would expose just a fraction of the root system proportional to the cover area. Thus,

$$D = H_r^{-1} \rho_{\text{veg}} \, \partial_t h \, \Theta(-\partial_t h) \tag{8}$$

where the length H_r can be interpreted as the ratio of the volume of the root system to its cover area, and characterizes plant sensitivity to erosion: higher values of H_r mean a more resilient vegetation as erosion exposes a lower fraction of the root system. In the model we set $H_r = 1\text{m}$.

1.6.3 Lateral vegetation growth

We call lateral vegetation growth the increase in cover at one place due to biomass production at neighboring places, mainly by underground rhizome growth. Hence, we assume the lateral growth rate to be proportional to the absolute value of the spatial variability (gradient) of the cover fraction $|\nabla \rho_{\text{veg}}|$

$$G_l = C |\nabla \rho_{\text{veg}}| \tag{9}$$

where the absolute value denotes isotropic growth and C is the rhizome growth rate. Assuming rhizome growth is stimulated by sand accretion and limited by the local slope $|\nabla h|$, we get in a first approximation:

$$C = \beta \, \partial_t h \, \Theta(\partial_t h) \, \Theta(\tan \theta_c - |\nabla h|) \tag{10}$$

Therefore, vegetation propagates laterally in areas no steeper than θ_c and with a rate dictated by the proportionality constant β. In the model we set $\beta = 100$ and $\theta_c = 15^o$.

1.7 Initial condition

1.7.1 Sand surface

The sand surface elevation h is initially defined as an inclined plane $h(x, y) = \tan(\alpha)\, x$ at the foreshore $(x < 0)$, and as a flat surface $h(x, y) = 0$ at the backshore $(x > 0)$, where α is the beach slope. In the model we set $\alpha = 1°$.

1.7.2 Vegetation

Because of the initial exponential growth, in the absence of lateral propagation, plants cannot grow in un-vegetated areas. Therefore, the growth rate equation needs an initial condition, defining a minimum value for the cover fraction to start plant colonization. We assume seeds or rhizome fragments are pervasive and thus the cover fraction is set to a small value $\rho_{\text{veg}}^{\text{min}}$ in all unvegetated areas instead of $\rho_{\text{veg}} = 0$. We set $\rho_{\text{veg}}^{\text{min}} = 10^{-4}$.

1.8   Boundary conditions

We assume no aeolian sand influx ($q_a = 0$) at the most seaward limit of the foreshore, where the surface elevation equals the mean low water level (MLWL) and thus the sand is effectively wet at all times and not available for aeolian transport.

1.9   Integration

We integrate the model within a two-dimensional domain large enough to include the resulting morphology. The grid spacing and time step are typically $\sim 1/4$m and $\sim 1/2$ hour, respectively, and are selected to resolve the smallest length and temporal scales involved in the problem, the saturation length l_{sat} and time $l_{\text{sat}}^2/q_{\text{sat}}$. For simplicity all boundary conditions are uniform in the alongshore direction y.

1.10   Parameters for simulations

We investigate model outcomes as a function of the parameters characterizing the external forcing (the explored range is in parenthesis): shoreline progradation rate $\dot{x}_{\text{shore}}$ (0-15 m/yr) and the imposed onshore wind, characterized by the ratio of the undisturbed shear velocity u_{*0} and the transport threshold u_t (1.5-2), which is assumed constant and oriented along the cross-shore direction throughout each simulation. Notice that by definition $u_{*0}/u_t = \sqrt{\tau_0/\tau_t}$ where τ_0 is the undisturbed bed shear stress and τ_t is the threshold. Simulation time is thus shorter than in more realistic wind conditions, where the wind intensity fluctuates daily and seasonally, beach sand could be wet (i.e. transport threshold becomes very large) or sand supply is limited. The conversion factor is loosely defined by the fraction of the time r_t the wind is above the transport

threshold and sand is available. Although the value $r_t = 0.1$ is typically used it is important to note that this parameter essentially rescales time for all processes involved.

2   Measured data

Location	Shoreline progradation rate (m/yr)	Vertical dune growth rate (m/yr) dune 1	dune 2	dune height (m) dune 1	dune 2	stoss slope
1	2.0 ± 1.1	0.46	0.46	5.5	3.3	0.10
2	3.0 ± 0.6	0.45	0.5	6.0	5.8	0.14
3	3.1 ± 0.7	0.45	0.49	6.1	6.0	0.17
4	3.4 ± 0.9	0.38	0.5	6.7	5.6	0.15
5	2.3 ± 0.7	0.52	0.47	6.9	6.2	0.13
6	1.5 ± 0.7	0.59	0.56	7.2	6.6	0.15
7	2.0 ± 0.8	0.56	0.47	6.6	5.5	0.17
8	2.9 ± 0.9	0.36	0.39	5.4	3.9	0.13
9	$3.5 \pm 1.$	0.46	0.23	5.2	4.0	0.07
10	3.3 ± 0.9	0.4	0.5	4.4	6.3	0.13
11	1.3 ± 0.7	0.5	0.43	6.7	4.5	0.17
12	2.7 ± 0.8	0.35	0.25	5.0	4.3	0.13

Table DR1: Values calculated from measured elevation profiles. Labels *dune 1* and *dune 2* refer to foredunes identified in 2012 and 2004's profiles, respectively.

Supplementary References

1. Durán, O., Silva, M.V.N., Bezerra, L.J.C., Herrmann, H.J. & Maia, L.P. Measurements and numerical simulations of the degree of activity and vegetation cover on parabolic dunes in north-eastern Brazil. *Geomorphology* **102**, 460–471. (2008).

2. Durán, O., Parteli, E.J.R. & Herrmann, H.J. A continuous model for sand dunes: Review, new developments and application to barchan dunes and barchan dune fields. *Earth Surf. Proc. Land.* **35**, 1591–1600. (2010).