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SUPPLEMENTAL INFORMATION

APPENDIX DR1

Barenblatt (1962) shows that the stress intensity factor K_0 for a fluid-filled crack of length l with a uniform fluid pressure $P_{f,0}$ within a homogeneous isotropic lithostatic stress field of magnitude σ_{lith} is given by

$$K_0 = \Delta P \sqrt{l/2}, \quad (DR1-1)$$

where $\Delta P = P_{f,0} - \sigma_{lith}$ is the net pressure. See also Barenblatt (2014, chapter 5) for an abbreviated analysis.

A tensile crack expands in a critical fashion if the stress intensity factor K_0 is equal to or exceeds a critical value K_c (Griffith's criterion), that is,

$$K_0 \geq K_c. \quad (DR1-2)$$

Following Barenblatt (1962), Aki et al. (1977) consider the following situation: the original crack is fully filled with fluid. During crack expansion the fluid front cannot keep up with the moving crack tip, creating a fluid lag zone, that is, an area at the crack tip which is free of fluids. The crack advances by a distance Δl , and the fluid front is assumed to be stationary for simplicity. The fluid-free zone is at zero pressure. Crack extension then creates a cohesive force, or negative stress intensity K_I , equal to

$$K_I \approx -0.9 \sigma_{lith} \sqrt{(\Delta l/2)}, \quad (DR1-3)$$

because of the zero fluid pressure within the fluid-free zone. After expansion, the stress intensity $K_{new,Aki}$ at the moved crack tip becomes

$$\begin{aligned} K_{new,Aki} &= (P_{f,0} - \sigma_{lith}) \sqrt{(l/2 + \Delta l/2)} - 0.9 \sigma_{lith} \sqrt{(\Delta l/2)} \\ &= K_0 + (P_{f,0} - \sigma_{lith}) \sqrt{(\Delta l/2)} - 0.9 \sigma_{lith} \sqrt{(\Delta l/2)}. \end{aligned} \quad (DR1-4)$$

This follows from equations (DR1-1) and (DR1-3) if the new crack length $l + \Delta l$ is taken into account. If crack extension occurs for $K_0 \approx K_c$, then Aki et al. (1977) argue that the presence of a fluid lag zone will halt crack extension since $K_{new,Aki} < K_c$ for a reasonable net pressure ΔP and lithostatic stresses at say more than 500m depth (around 10MPa). Their analysis implies that renewed crack growth then only happens once the fluids have reached the crack tip or the fluid

pressure P_f inside the crack is increased, thus countering the negative stress intensity due to the fluid lag zone.

Aki et al. (1977) argue that the above analysis implies that only continuous fracture growth is possible, since any crack extension Δl , even an infinitesimally small one, leads to halting. In this scenario, the fluid speed becomes equal to the rupture velocity, a common assumption in mathematical models for hydraulic fracture propagation (Adachi et al., 2007). However, Griffith's criterion, equation DR1-2, only predicts when tensile failure occurs. It does not determine the fracture extension length Δl . Likewise, the Mohr-Coulomb failure criterion for shearing only predicts if failure is likely, but not the rupture length. Dynamic fracture models are required to predict parameters such as rupture (fracture) length, area and displacement (that is, slip and/or aperture).

For instance, the seismic moment M_0 for shear failure is given by

$$M_0 = (16/7) \Delta\sigma_s a^3, \quad (DR1-5)$$

where $\Delta\sigma_s$ is the static stress drop and a is the fracture radius (Eshelby, 1957; Keylis-Borok, 1959; Walter and Brune, 1993). Equation DR1-5 holds for a Poisson's body (that is, equal Lamé parameters or $\mu = \lambda$). The static stress drop $\Delta\sigma_s$ is defined as the difference between shear stress across the fault before and after the event. A stress drop causes a reduction in driving stress. Shear failure halts when the shear stress becomes less than the frictional stress and cohesive forces. Equation DR1-5 shows that the seismic moment M_0 and thus the size of an event is proportional to the stress drop $\Delta\sigma_s$, which in turn is determined by the material and stress heterogeneity.

In a similar fashion, it can be shown that the seismic moment M_0 for a tensile event is equal to

$$M_0 = \sqrt{22} \Delta P_s a^3, \quad (DR1-6)$$

where ΔP_s is the static fluid pressure drop and fracture radius $a = \Delta l/2$. Again a Poisson's body is assumed. Equation DR1-6 is derived using the following relationships: (1) the nonzero moment tensor components for a tensile crack in the horizontal plane and opening along the vertical axis, are $M_{11} = M_{22} = \lambda AD$ and $M_{33} = (\lambda + 2\mu)AD$, with $A = \pi a^2$, the surface area, and D is average displacement (aperture); (2) the moment M_0 is defined as $M_0 = \sqrt{(M_{ik}M_{ik}/2)}$ with summation over all indices i and k of the full moment tensor (Julian et al., 1998); and (3) the average aperture D

$= 2a \Delta P_s / (\pi \mu)$ (Sneddon, 1951; Walter and Brune, 1993; Eaton et al., 2014b). Tensile cracks oriented in other directions yield the same result.

Note that in analogy with equation DR1-5, variable ΔP_s represents a static stress drop, whereas Walter and Brune (1993) and Eaton et al. (2014b) assume it is equal to the effective net pressure.

If the argument of Aki et al. (1977) that any crack extension, including an infinitesimally small one, leads to halting is applied to the Mohr-Coulomb failure criterion, it predicts that only slow slip is possible but no abrupt shear failure (and thus no felt seismicity, equation DR1-5), since it assumes a medium with homogeneous material properties (e.g., no asperities) and homogeneous stresses. Likewise, stress and material heterogeneity ensures episodic tensile crack growth where the seismic moment M_0 of individual events is given by equation DR1-6. For instance, in our numerical simulations the presence of natural fracture sets and intact rock bridges ensures stress and material heterogeneity. It is important to note that variations in material properties generally tend to cause stress heterogeneity [e.g., Roche and Van der Baan (2015)].

The above analysis thus supports episodic instead of continuous crack propagation for hydraulic fracturing treatments which are often around 2km depth, targeting heterogeneous rocks such as shales. Furthermore, the negative stress intensity, equation (DR1-3), also leads to partial closure of the crack near its tip, creating a process reminiscent of hand clapping. Continuous and possibly even accelerated fracture growth is however possible at shallow depths since the lithostatic stress σ_{lith} becomes then small, reducing the effect of the closing negative stress intensity, equation (DR1-3).

Secor (1969) invokes a related but different reasoning to argue for episodic crack growth. In his model, crack extension leads to additional crack volume, temporarily reducing the fluid pressure P_f at the crack tip, thus lowering the stress intensity factor. Specifically the new fluid pressure $P_{f,new}$ changes from the original fluid pressure $P_{f,o}$ as

$$P_{f,new} = P_{f,o} - c^{-1} \Delta V / V, \quad (DR1-7)$$

with c the fluid compressibility coefficient and $\Delta V / V$ the relative increase in crack volume (not necessarily uniform across the entire crack). For a perfect gas $c^{-1} \approx P_{f,o}$. Therefore, a 10% increase in volume leads to a 10% reduction in fluid pressure near the crack tip. For water, $c^{-1} = 1.96 \times 10^9$ Pa, and a 1% increase in volume corresponds to a 19.6MPa reduction in fluid pressure. This is

substantial if we consider for instance a reservoir around 2km depth with 10MPa net pressure ($P_{f,o} = 60\text{MPa}$, $\sigma_{lith} = 50\text{MPa}$).

The original stress intensity factor K_0 prior to crack extension is again given by equation (DR1-1). After crack extension the new stress intensity factor $K_{new,Secor}$ then becomes

$$\begin{aligned} K_{new,Secor} &= (P_{f,o} (1 - \Delta V/V) - \sigma_{lith}) \sqrt{l/2 + \Delta l/2} \\ &= K_0 + (P_{f,o} - \sigma_{lith}) \sqrt{\Delta l/2} - (c^{-1} \Delta V/V) \sqrt{l/2 + \Delta l/2}. \end{aligned} \quad (DR1-8)$$

If we assume again that crack extension happens when $K_0 \approx K_c$, then the reduction in fluid pressure at the crack tip is likely to cause halting of the crack propagation since $K_{new,Secor} < K_c$ due to the negative third term in equation (DR1-8). Crack propagation continues when new fluids have reverted the fluid pressure within the additional volume ΔV to the original fluid pressure $P_{f,o}$, and the third term becomes close to zero again. In addition, if the new fluid pressure $P_{f,new}$ becomes less than the lithostatic stress σ_{lith} then the net pressure ΔP at the crack tip becomes negative, causing again partial crack closure, as indicated by equation (DR1-1).

Equation (DR1-8) implies there is a trade-off between increased stress intensity due to the increased crack length and reduced stress intensity due to volume increase and thus drop in fluid pressure at the crack tip. In practice, the third term in equation (DR1-8), reduced stress intensity due to a local pressure drop, is likely to be somewhat tempered since (1) the height of most hydraulic fractures is substantially smaller than their length, whereas the above analysis assumes a crack that is infinite in one direction, and (2) the pressure drop only occurs at the crack tip, not across the entire crack length. A constricted height limits the total moment (that is, force times crack length) exerted on the crack tip, thus reducing the effective crack length. Likewise, the effective crack length is reduced since only the portion where the pressure drop occurs should be taken into account. Nonetheless, considering some representative numbers such as a reservoir around 2km depth with 10MPa net pressure ($P_{f,o} = 60\text{MPa}$, $\sigma_{lith} = 50\text{MPa}$) and a 5m effective fracture that extends by 1 meter ($l=5\text{m}$, $\Delta l=1\text{m}$) with merely a 1% increase in volume ($\Delta V/V = 0.01$) demonstrates the third term is generally dominant. The importance of the second term increases with decreasing depth, that is, decreasing lithostatic stresses, , pointing again at the possibility of continuous fracture growth at shallow depths.

Aki et al. (1977) and Secor (1969) invoke different causal mechanisms, yet both analyses predict episodic instead of continuous crack growth for anthropogenic hydraulic fracturing treatments at common depths due to reduced fluid pressures at the crack tip. Secor's (1969)

model does not involve a fluid lag zone, that is, a fluid-free zone at the crack tip. A fluid lag zone has been seen in laboratory investigations (Medlin and Masse, 1984; Groeneboom et al., 2003). The size of the fluid lag zone is however unclear and subject to theoretical investigations (Garagash and Detournay, 2000); yet its existence will encourage temporal halting of fracture propagation due to a local pressure drop with associated partial closing.

Continuous fracture growth is often inferred in case of natural hydraulic fracturing (Bahat and Engelder, 1984; Savalli and Engelder, 2005). The above analysis points to several conditions under which this might occur at critical stress intensities, including very high net pressures ΔP , shallow depths and thus small lithostatic stresses σ_{lith} , fracturing due to inflow of gas which is highly compressible, growth into mechanically weak zones or into areas with low effective stresses (that is, due to elevated pore pressures or reduced lithostatic stresses, for instance, because of upward growth), and finally a reduction in critical stress intensity K_c during fracture propagation, equations (DR1-2), (DR1-4) and (DR1-8). See also Lacazette and Engelder (1992) who discuss the effect of fluid infiltration into the hydraulic fracture, compressibility of the fluid inside the hydraulic fracture, and the effect of fluid flow velocities (in particular, Secor's 1969 model) on episodic versus continuous fracture growth at critical stress intensities.

A reduction in critical stress intensity K_c during tensile fracture propagation would imply the existence of a larger, static and a smaller, dynamic critical stress intensity for tensile failure, analogous to static and dynamic shear friction coefficients governing shear failure (Marone, 1998; Rubinstein et al., 2004). Tests however show that the dynamic tensile strength of rocks (measured at high strain rates) is larger than the quasi-static strength (Rubin and Ahrens, 1991), likely due to stress redistribution within samples and crack arrests because of increasing microcrack densities with increasing strain rates (Cho et al., 2003). The mathematical analysis thus confirms the numerical results of stick-split behavior and repeated opening and closing (hand clapping) near anthropogenic hydraulic fracture tips.

APPENDIX DR2

Far-field body wave spectra radiated from a small circular crack can be represented using a variant of the Brune source model (Walter and Brune, 1993). Eaton et al. (2014b) analyzed microseismic data from a hydraulic fracturing treatment targeting the Montney formation in British Columbia, Canada (Eaton et al., 2013). Their Figure 13 shows one microseismic event displaying spectral notching. We analyzed microseismic events that occurred during a fracturing treatment into a tight-sand formation in Alberta, Canada at ~1860m depth (Eaton et al., 2014a). The microseismic events were recorded using a 12-level triaxial geophone array installed in a vertical monitor well, in a depth range from 1606m to 1835 m.

The procedure used to process each microseismic dataset is similar to Oye and Roth (2003) and includes the following steps: 1) application of an instrument-response correction to convert recorded signals into units of displacement; 2) rotation of 3-component traces into ray-centered co-ordinates, thereby isolating P, S_{fast} and S_{slow} signals; 3) stacking of P, S_{fast} and S_{slow} waveforms by beam forming using an iterative cross-correlation approach with static shifts and polarity checking, similar to the method of De Meersman et al. (2009); 4) calculation of P and S displacement spectra by computing the Fourier transform of the recorded waveforms within a 250ms window centered on the body-wave arrival, where anisotropy of the radiated S wave spectrum is accounted for using the vector sum of the S_{fast} and S_{slow} signals. The noise spectrum is extracted from a 250ms pre-event window. The best-fitting Brune spectrum and clapping spectrum are obtained using a least-squares fitting approach (Eaton et al., 2014b), which is parameterised based on quality factor (Q) for P and S waves, corner frequency, low-frequency plateau amplitude and time interval τ between opening and closing. The S/P amplitude ratio is measured using the fitted low-frequency plateau amplitudes for P and S waves.

Following Eaton et al. (2014b), the tensile crack radius a is estimated using

$$\log_{10}(a) = 3.05 - (1/6)\log_{10}(22) + M_w/2 - (1/3)\log_{10}(\Delta P_s), \quad (\text{DR2-1})$$

where ΔP_s is the static fluid pressure drop, equation DR1-6, which for simplicity we take to be equal to the net pressure ΔP (fluid pressure minus minimum principal stress, equation DR1-1). The latter is the pressure that is required to keep the fracture open. Equation DR2-1 is derived by taking the logarithm of equation DR1-6 and combining this with $M_w = (2/3)\log_{10}(M0)-6.1$.

The tensile aperture E can be obtained from equation (3) in Walter and Brune (1993), yielding

$$E = 3a \Delta P / (\pi \mu), \quad (\text{DR2-2})$$

with μ the shear modulus of the intact rock.

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Table DR1: Parametric inputs used in the numerical model for stick-split verification, based on in situ conditions in an underground mine in New South Wales, Australia, after Preisig et al., (2015).

Natural fracture network properties				
		Dip direction [degrees]	Dip angle [degrees]	Spacing [m]
Family 1 (only natural fractures)		090	81	2.4
Family 2 (only natural fractures)		270	63	3.6
Family 3 (natural and incipient fractures)		270	15	4.2
Persistence of fractures: fully or variable (see Fig. 3)				
Rock properties		Fractures properties		
Young modulus E [Pa]	$60 \cdot 10^9$		Incipient	Natural
Poisson ratio ν [-]	0.25	Normal stiffness k_n [Pa/m]	$1.3 \cdot 10^{11}$	$1.3 \cdot 10^{11}$
Density ρ [kg/m ³]	2700	Shear stiffness [Pa/m]	$1.3 \cdot 10^{10}$	$1.3 \cdot 10^{10}$
		Tensile strength T [Pa]	$-0.5 \cdot 10^6$	0.0
		Cohesion C [Pa]	$1.0 \cdot 10^6$	0.0
Fluid properties and constants		Frictional angle Φ [°]	30	45
Viscosity μ [Pa s]	0.001	Aperture at zero effective	$2.0 \cdot 10^{-5}$	$2.0 \cdot 10^{-5}$
Density ρ_w [kg/m ³]	1000	normal stress a_0 [m]		
Bulk modulus K_w [Pa]	$0.1 \cdot 10^9$	Residual aperture a_{res} [m]	$4.0 \cdot 10^{-6}$	$4.0 \cdot 10^{-6}$
Gravity g [m/s ²]	9.81	Dilation angle Ψ [°]	20	20
In-situ stress state				
Stress σ [MPa] at depth: 1415 [m]		Orientation		
$\sigma_1 = 73$		Horizontal E-W		
$\sigma_3 = 42$		Vertical		