APPENDIX: UPSCALING OF EXPERIMENTAL TO NATURAL TURBIDITY CURRENTS

This appendix uses Similarity Laws to show how the experimental intrabed turbidity currents presented in this paper scale to natural turbidity currents in general terms.

Froude number similarity and sediment type similarity

Similarity Laws are used frequently for small-scale laboratory simulations of dam breaks, flows in ports and other types of earth-surface flow. Based on dimensional analysis of sediment gravity flow dynamics (*e.g.*, Middleton, 1966; Peakall *et al.*, 1996), it has been proposed that the scale model and prototype adhere to densiometric Froude number (*Fr'*) similarity, whilst the scale model has a sufficiently high Reynolds number (*Re* > 3,000) to maintain turbulent particle support. For the sake of simplicity, we apply this analysis to the experimental turbidity currents before these flows started to interact with the soft muddy substrate. Table DR1 shows *Re*-values between 6,270 and 13,020 for the present experiments, which is well above the threshold for turbulent flow.

Froude number similarity can be verified using the following physical parameters for the simulations presented in this paper:

1. Flow velocity, *U*: In the scale model, flow velocity ranged from 0.06 $m \cdot s^{-1}$ to 0.12 $m \cdot s^{-1}$, from which we adopt a typical value of 0.09 $m \cdot s^{-1}$. Estimated flow velocities of natural turbidity currents are of the order of 10 $m \cdot s^{-1}$ or less. Here, we adopt an average flow velocity of 5 $m \cdot s^{-1}$, calculated from Table 1 of Talling *et al.* (2013), although it should be emphasized that the data spread around this velocity is large.
2. Flow density, ρ_f: Coal suspended in water at volumetric concentrations of 1-23% (Table 1) is equivalent to densities of 1002-1044 $kg \cdot m^{-3}$. Natural flows, laden with quartz-rich sediment particles instead of coal, have densities of 1017-1396 $kg \cdot m^{-3}$ at the same range of volumetric concentrations.
3. Flow thickness, *H*: The flows simulated in the laboratory experiments were approximately 0.10 m thick, while natural flows have been found to be thicker by 3 order of magnitude, *i.e.* 100 m (*cf.* Table 1 of Talling *et al.*, 2013).

The ratio of the thickness of the model flows and the typical thickness of natural flows thus provides the vertical scale factor, E_{vert} = 1 : 1,000.

The length of the flume used in the present experiments is 5 m. Soft muddy substrates are more likely to be stable on the continental rise than on the continental slope, as these substrates might become unstable at the average slope gradient of 3° on the continental slope. Submarine fans formed by sand-rich turbidity currents on the continental rise are typically 10-50 km long and these fans tend to lack longitudinal separation of channels and lobes (Reading and Richards, 1994). Taking an average channel length of 25 km gives a horizontal scale factor between model and prototype, E_{hor} = 1:5,000.

The scale model distortion factor, φ, is defined as:

$$\varphi = \frac{E_{hor}}{E_{vert}} \qquad (1)$$

which is equal to 5 for the present experiments, and thus just below the maximum value of 5 to 10, recommended by Chanson (1999).

The densiometric Froude number is given by:

$$Fr' = \frac{U}{\sqrt{g' \cdot H}} \qquad (2)$$

where *g′* is the density-reduced gravity constant:

$$g' = g \cdot \left(\frac{\rho_f - \rho_a}{\rho_a} \right) \qquad (3)$$

where ρ_a is the density of the ambient fluid, ρ_f is the flow density, and *g* is the constant due to gravity. The density of the turbidity current is related to the volumetric concentration of suspended sediment, C_v, by:

$$\rho_f = \rho_a + (\rho_s - \rho_a) \cdot C_V \qquad (4)$$

where ρ_s is the sediment density. Combining Equations 3 and 4 yields:

$$g' = g \cdot (\beta_s - 1) \cdot C_V \qquad (5)$$

where $\beta_s = \rho_s / \rho_a$.

Froude number similarity requires that (*m* = model flow, *n* = natural flow):

$$Fr'_m = Fr'_n \qquad (6)$$

or with Equation 2:

$$\frac{U_m^{\ 2}}{g'_m \cdot H_m} = \frac{U_n^{\ 2}}{g'_n \cdot H_n} \qquad (7)$$

and with Equation 3:

$$\frac{U_m^{\ 2}}{g_m \cdot H_m \cdot (\beta_s - 1)_m \cdot C_{V_m}} = \frac{U_n^{\ 2}}{g_n \cdot H_n \cdot (\beta_s - 1)_n \cdot C_{V_n}} \qquad (8)$$

and solving for velocity:

$$\frac{U_m^{\ 2}}{U_n^{\ 2}} = \frac{g_m}{g_n} \cdot \frac{(\beta_s - 1)_m}{(\beta_s - 1)_n} \cdot \frac{C_{V_m}}{C_{V_n}} \cdot \frac{H_m}{H_n} \qquad (9)$$

Considering that $g_m = g_n$ and defining $\aleph = \frac{(\beta_s - 1)_m}{(\beta_s - 1)_n}$, the following horizontal velocity scale factor, E_{vel_hor}, can be derived:

$$E_{vel\_hor} = \sqrt{\frac{U_m^{\ 2}}{U_n^{\ 2}}} = \sqrt{\aleph \cdot \frac{C_{V_m}}{C_{V_n}} \cdot \frac{H_m}{H_n}} = \sqrt{\aleph \cdot E_{C_v} \cdot E_{vert}} \qquad (10)$$

where $E_{C_v} = C_{V_m}/C_{V_n}$ and $E_{vert} = H_m/H_n$.

Assuming scale model and prototype flows of equal density, $E_{C_v} = C_{V_m}/C_{V_n} = 1$, Equation 10 reduces to:

$$E_{vel\_hor} = \sqrt{\aleph . E_{vert}} \qquad (11)$$

The density of the coal used in the present study is 1,190 kg·m^{-3}, while the density of natural sediment typically approaches the density of quartz, ρ_s = 2,650 kg·m^{-3}. Hence:

$$\aleph = \frac{(\beta_s - 1)_m}{(\beta_s - 1)_n} = \frac{(1.19 - 1)}{(2.57 - 1)} = 0.121 \qquad (12)$$

Inserting Equation 12 into Equation 11, and using E_{vert} = 0.001, yields:

$$E_{vel\_hor} = \sqrt{\frac{U_m^{\ 2}}{U_n^{\ 2}}} = 0.011 \qquad (13)$$

or, with U_m = 0.09 m·s^{-1}:

$$U_n = 8.18 \ \mathrm{m\,s^{-1}} \qquad (14)$$

This U_n-value is close to the average velocity of 5 m·s^{-1} derived from Talling et al. (2013). We therefore conclude that, based on Froude number similarity, the scale model flows satisfactorily compare to the prototype flows.

Sediment type similarity can be determined using the following fundamental scaling relationship (Valembois, 1960; Le Méhauté, 1970):

$$\frac{X}{Z} = \frac{U}{W} \qquad (15)$$

where, X is the horizontal length scale, Z is the vertical length scale, and W is the vertical velocity scale. Equation 15 can be expressed in terms of model-to-prototype scale ratios as:

$$\varphi = \frac{E_{hor}}{E_{ver}} = \frac{E_{vel\_hor}}{E_{vel\_ver}} \qquad (16)$$

where E_{vert_vel} is the ratio of fall velocity of particles in the model, W_m, to that of the prototype, W_n, and φ is the distortion factor (here, $\varphi = 5$). Using $E_{vel\_hor} = 0.011$ (Equation 13), Equation 16 can be rewritten as:

$$E_{vert\_vel} = \frac{W_m}{W_n} = 0.055 \qquad (17)$$

Here, Stokes law is used as a first-order approximation for particle fall velocity:

$$W = \frac{D^2 \cdot g \cdot (\rho_s - \rho_f)}{18 \cdot \mu} \qquad (18)$$

where D is the diameter of the suspended sediment, and μ is the dynamic viscosity of the ambient fluid.

Combining Equations 17 and 18 yields:

$$\frac{D_m{}^2 \cdot g_m \cdot (\rho_s - \rho_a)_m}{18 \cdot \mu_m} = 0.055 . \frac{D_n{}^2 \cdot g_n \cdot (\rho_s - \rho_a)_n}{18 \cdot \mu_n} \qquad (19)$$

and solving for the diameter of suspended sediment:

$$\frac{D_m{}^2}{D_n{}^2} = 0.055 \cdot \frac{\mu_m}{\mu_n} \cdot \frac{g_n}{g_m} \cdot \frac{(\rho_s - \rho_a)_n}{(\rho_s - \rho_a)_m} \qquad (20)$$

Considering that $g_m = g_n$, and assuming that the viscosities of the model flow and prototype are identical, Equation 20 reduces to:

$$E_{vel\_vert} = \frac{D_m}{D_n} = 0.2345 \cdot \sqrt{\frac{(\rho_s - \rho_a)_n}{(\rho_s - \rho_a)_m}} \qquad (21)$$

Inserting the densities of coal for the model fresh water flows and quartz for the prototype seawater flows (*cf.* Equation 12), yields:

$$E_{vel\_vert} = \frac{D_m}{D_n} = 0.2345 \cdot \sqrt{\frac{2650-1030}{1190-1000}} = 0.6848 \qquad (22)$$

The diameter of the coal, D_m, used in the simulations ranged from 0.03 mm to 0.3 mm. In Equation 22 this corresponds to D_n-values of 0.044 mm and 0.44 mm, respectively. We therefore conclude that, based on sediment type similarity, the scale model flows compare to natural turbidity currents that carry coarse silt to medium sand particles. This is reasonable at flow velocities of 8 m s^{-1} (Equation 14).

Summary

The experimental turbidity currents, before showing intrabed behavior, are approximately equivalent to *c.* 100 m thick natural turbidity currents that move at *c.* 8 m s^{-1} and carry silt- and sand-sized quartz-rich sediment.

Table DR1. Experimental Parameters

Run	C_o (vol %)	ρ_f (kg m^{-3})	U_{mud} (mm s^{-1})	H (mm)	Re_{mud} (-)	U_{fixed} (mm·s^{-1})	IP (%)
1	1	1002	62	120	7440	40	0
2	5	1010	93	140	13020	60	41
3	10	1019	117	100	11700	74	84
4	15	1029	119	75	8925	132	100
5	23	1044	114	55	6270	146	100

C_o = initial coal concentration in turbidity current
ρ_f = initial density of turbidity current
U_{mud} = mean head propagation velocity of turbidity current over muddy substrate at x=0.12-0.34 m
H = thickness of head of turbidity current at x=0.25 m
Re_{mud} = Reynolds number, based on U_{mud}, H and water viscosity
U_{fixed} = mean head propagation velocity of control current over fixed substrate at x=0.12-0.34 m
IP = proportion of intrabed flow at front of turbidity current
Note: Coal was used as the light-weight, non-cohesive sediment driving the experimental turbidity current. In natural turbidity current, this sediment most commonly consists of siliciclastic sand.

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