1 Henry's Law

Henry's law calculation for outgassing of pressurized water during subglacial earthquake.

Consider a cavity of initial volume V, completely filled with water saturated with nitrogen, that expands by

$$dV = dV_l + dV_g \tag{1}$$

during an earthquake, with the subscripts indicating liquid and exsolved gas.

From Henry's Law, the volumetric concentration of nitrogen molecules in the water is $C = P/K$ with P the pressure and K the Henry's-law constant ([*Mackay and Shiu*(1981)]). The number of nitrogen molecules in the water is then $N = PV/K$. Noting that $dV_l << V$, the number of molecules exsolving into a gas phase during the quake, N_g, is the difference between those in the nearly-unchanged volume of water V at the initial pressure P_1 and the final pressure P_2,

$$N_g = V(P_1 - P_2)/K \tag{2}$$

Then, using the gas law at absolute temperature T with gas constant R,

$$dV_g = N_g RT/P_2 \tag{3}$$

The water has compressibility β, so

$$dV_l = \beta V(P_1 - P_2) \tag{4}$$

Substituting into equation 1 from equations 3 and 4 yields

$$dV = \beta V(P_1 - P_2) + N_g RT/P_2 \tag{5}$$

and substituting for N_g from Equation 2

$$dV = \beta V(P_1 - P_2) + \frac{V(P_1 - P_2)RT}{KP_2} \tag{6}$$

Multiplying by P_2 and dividing by V and β gives

$$\frac{P_2 dV}{\beta V} = -P_2^2 + P_1 P_2 - P_2\left(\frac{RT}{\beta K}\right) + P_1\left(\frac{RT}{\beta K}\right) \tag{7}$$

Then, adding or subtracting to move everything to the left-hand side, and grouping in terms of P_2 gives

$$P_2^2 + P_2\left(\frac{dV}{\beta V} - P_1 + \frac{RT}{\beta K}\right) - \frac{RT}{\beta K}P_1 = 0. \tag{8}$$

which is the quadratic for the final pressure P_2. The solution with physically possible, positive pressure is

$$P_2 = \frac{-\left(\frac{dV}{\beta V} - P_1 + \frac{RT}{\beta K}\right) + \left[\left(\frac{dV}{\beta V} - P_1 + \frac{RT}{\beta K}\right)^2 + \frac{4RTP_1}{\beta K}\right]^{0.5}}{2} \tag{9}$$

The constants are $K = 10^5$ m^3 Pa^{-1} mol^{-1} ([*Rettich et al.*(1984)*Rettich, Battino, and Wilhelm*]), $\beta = 5 \times 10^{-10}$ Pa^{-1} ([*Fine and Millero*(1973)]), and $R = 8.3$ J K^{-1} mol^{-1}. Temperature will be close to or slightly below $T = 273$ K, depending on pressure.

For air-free water, our specified strain of $dV/V = 5 \times 10^{-3}$ would drop the pressure from 10 MPa to 0, sufficient to cause cavitation beneath about 1100 m of ice, but the pressure would drop only to 8 MPa if complete equilibrium were achieved starting from water fully saturated with nitrogen.

References

[*Fine and Millero*(1973)] Fine, R. A., and F. J. Millero (1973), Compressibility of water as a function of temperature and pressure, *The Journal of Chemical Physics*, *59*(10), 5529.

[*Mackay and Shiu*(1981)] Mackay, D., and W. Y. Shiu (1981), A critical review of Henry's law constants for chemicals of environmental interest, *J. Phys. Chem. Ref. Data*, *10*(4), 1175–1199.

[*Rettich et al.*(1984)*Rettich, Battino, and Wilhelm*] Rettich, T., R. Battino, and E. Wilhelm (1984), Solubility of gases in liquids. XVI. Henry's law coefficients for nitrogen in water at 5 to 50 C, *Journal of solution chemistry*, *13*(5), 335–348.