METHODS SUMMARY

The time series from HOLV-G was expanded into the time-frequency space (Fig. 2B) using the Continuous Wavelet Transform (CWT; Grinsted et al., 2004), which is preferable to the Fourier Transform because the former allows a more accurate time-frequency description of signals covering a wide frequency range (Daubechies, 1990). The CWT is particularly suited to extracting localized intermittent periodicities, even in low signal-to-noise ratio conditions (Grinsted et al., 2004). It performs the convolution operation of the signal and the mother function, from which all the wavelet functions, used in the transformation, are derived, through translation (shifting) and scaling (dilation or compression) operations. We used the Morlet wavelet as the mother function.

A correlation analysis between the signals from the two gravity stations was carried out by calculating the Wavelet Coherence (WTC) between the CWTs of the two time series (Fig. 2C). This calculation finds localized correlation coefficients in time-frequency space and also quantifies the phase relationship between the two CWTs. Because of the different sampling rate at the two gravity stations, the signal from HOLV-G had to be resampled from 1 to 0.1 Hz before performing the WTC.

Both the CWT and WTC analyses were performed using the Matlab software package that can be found at http://www.pol.ac.uk/home/research/waveletcoherence/.

Coherence and admittance functions in the frequency domain between the signals from the HOLV-G and UWEV-G stations (Fig. 3) were calculated using the Matlab signal-processing tool, which is based upon Welch's averaged periodogram method (Welch, 1967). The coherence function (indicating how well the signal from one station corresponds to the signal from the other, at each frequency) is given by:

$$Cxy = \left(|Pxy|^2\right) / \left(Pxx\ Pyy\right),$$

where *Pxy* represents the cross spectral density, while *Pxx* and *Pyy* represent the power spectral densities. The admittance estimates the transfer function of the two signals using Welch's averaged periodogram method and is given by:

$$Txy = Pxy / Pxx\,.$$

The possible source locations that would result in an amplitude ratio of 4 between the signals from the two continuous gravity stations (coloured lines in the inset of Fig. 1) were calculated assuming an equal-dimensional gravity source (sphere). The gravity anomaly caused by a sphere at a point which is the origin of the local coordinate system is given by (Torge, 1989):

$$\Delta g = \frac{4}{3}\pi r^3 G \Delta\rho \frac{d}{\left(x^2 + y^2 + d^2\right)^{3/2}},$$

where r is the radius of the sphere, G is the Universal Gravitational Constant ($6.672*10^{-11}$ $m^3\ kg^{-1}\ s^{-2}$), $\Delta\rho$ is the density change, d is the depth of the sphere mass centre with respect to the elevation of the observation point. y and x define couples of planar coordinates corresponding to a possible position of the source which allows a to equal a fixed value (4, in our case). We assume that HOVL-G lies at the origin of the local coordinate system (0,0) and UWEV-G is D away from HOVL-G (D, 0). Thus, the ratio between the gravity effects at the two stations is given by:

$$a = \frac{\left[(x-D)^2 + y^2 + d_2{}^2\right]^{3/2} d_1}{\left(x^2 + y^2 + d_1{}^2\right)^{3/2} d_2},$$

where d_1 and d_2 are depth of the source mass centre with respect to the elevation of HOVL-G and UWEV-G, respectively. Substituting and simplifying, the above equation results in:

$$y = \sqrt{\frac{(x-D)^2 + d_2{}^2 - b\left(x^2 + d_1{}^2\right)}{(b-1)}},$$

where b is equal to $(ad_2 / d_1)^{2/3}$.

Since analytical models do not allow for density changes with different magnitudes throughout the source body, the amplitude of the “density inversion”, needed to induce the gravity changes observed at the surface, was calculated using a numerical model of the Halema’uma’u magma reservoir. The latter consists of 896 small elements, arranged in an equal-dimensional shape (Fig. 4B). The number of elements was chosen as a fair compromise between (i) a relatively small number of elements, enhancing calculation speed and (ii) the necessity of keeping the elements’ size much smaller than the distance between the observation point and the source (for the point approximation to hold). The elements (equal-dimensional and identical to each-other) can undergo different density changes and, since their distance to the observation point is always sufficiently larger than their size, the gravity effect that each of them produces can be safely approximated by the effect of an homogeneous sphere of the same mass. The overall gravity effect at the observation point is assessed by summarizing the effects of all the small elements:

$$\Delta g = GV \sum_{i=1}^{896} \left(\frac{\Delta\rho_i z_i}{\left(x_i{}^2 + y_i{}^2 + z_i{}^2\right)^{3/2}} \right),$$

where G is the Universal Gravitational Constant, V is the volume of each small element, $\Delta\rho_i$ is the density change undergone by the i-th small element, while (x_i y_i z_i) specifies the coordinates of the i-th element (the observation point is the origin of the local coordinate system).

REFERENCES

Daubechies, I., 1990, The wavelet transform, time-frequency localization and signal analysis: IEEE Transactions on Information Theory, v. 36, p. 961 - 1005, doi:10.1109/18.57199
Torge, W., 1989, Gravimetry: Berlin and New York, Walter de Gruyter, 465 p.

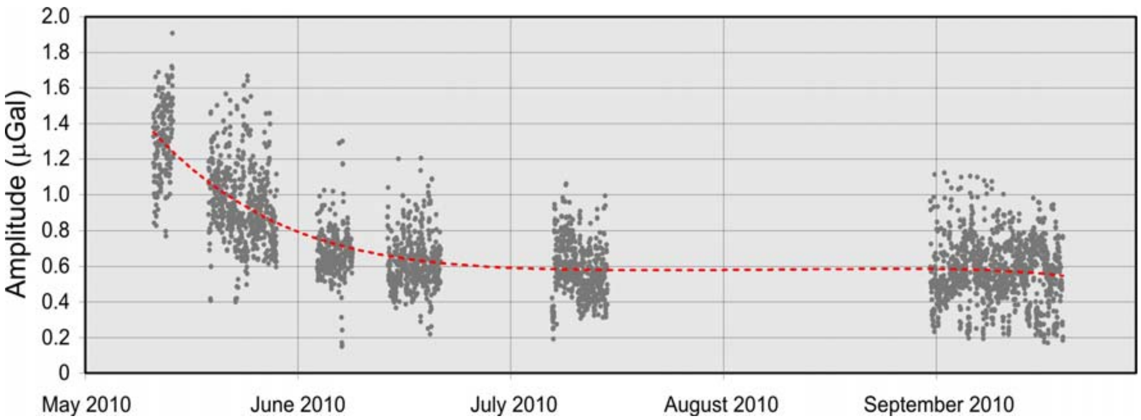

Figure DR1 - Amplitude of the ~0.0067 Hz gravity component over time at HOVL-G station. The average amplitude of the available data, between May and September 2010, is calculated over a 50 minute sliding window (50% overlap). The dashed line is a 4th degree polynomial fit of the data.