METHODS

The conduit flow models are based on the approach outlined in Colllier and Neuberg, 2006, with several notable updates. Conduit flow is solved through the use of the commercial finite element modelling package COMSOL Multiphisics®. The models are created in an axial symmetric domain space to minimise computing requirement, but allow the analysis of three dimensional effects. The mesh has a maximum node spacing of 7.5 m, with a layer of four boundary elements of 0.2 m thickness against the conduit wall. The conduit has a radius *r*, with the horizontal and vertical axis of the conduit (r,z) being aligned parallel to the horizontal and vertical axis of the model space.

Governing Equations

Conduit flow is solved with a finite element approach, and modelled in an axial symmetric domain space through the compressible formulation of the Navier-Stokes equation:

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nabla\cdot\left(\eta_s(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \frac{2}{3}\eta_s(\nabla\cdot\mathbf{u})\mathbf{I}\right) + \mathbf{F} \quad , \qquad \text{(DR1)}$$

and the continuity equation:

$$\frac{\partial p}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0 \quad , \qquad \text{(DR2)}$$

where ρ is density, $\mathbf{u}$ the velocity vector, p the pressure, η the dynamic viscosity, and $\mathbf{F}$ the volume force vector (gravity).

Magma Composition

The properties of the magma are modelled as the averaged properties of the magma constituents, melt, crystals and gas. Crystal content (χ_c) is assumed fixed at 30% vol. with a constant crystal density (ρ_c) of 2680 $\mathrm{Kgm^{-3}}$. Crystal growth is not considered. The expression for the bulk density of the magma is given by:

$$\rho = \left(\rho_m\chi_m(1-\chi_g) + (\rho_g\chi_g) + \left(\chi_c\rho_c(1-\chi_g)\right)\right) \quad , \qquad \text{(DR3)}$$

where χ_m is the initial fraction of melt (70% vol.) and ρ_m is the assumed melt density (2300 kg $\mathrm{m^{-3}}$). For the gas phase, water is assumed as the only volatile species present at an initial concentration of 4 wt% (Barclay et al., 1998). The gas density (ρ_g) is calculated from the ideal gas law:

$$pV = n\mathrm{R}T \quad , \qquad \text{(DR4)}$$

where V is the volume of gas, R the ideal gas constant and T the temperature. The number of moles of water, n, is related to density by:

$$n = \mathrm{M}/m \ , \quad \text{(DR5)}$$

where M is the molar mass of water and m is the mass of water present. Thus, combining (7) and (8) and setting V to 1 m^3, we get:

$$\rho_{\mathrm{g}} = \frac{mp}{\mathrm{R}T} \ . \quad \text{(DR6)}$$

Magma temperature is taken as 1100 K (Devine et al., 2003) with a thermal boundary layer (TBL) of 0.4 m defined adjacent to the conduit wall. A linear temperature drop of 200 K (Colllier and Neuberg, 2006) is applied across the TBL, which is included to simulate the cooling of the magma abutting the country rock in a well established conduit. The gas volume fraction (χ_g) is calculated by determined how much water remains dissolved within the melt at a particular pressure (Liu et al., 2005). At high enough pressures, all the water is dissolved within the melt fraction and χ_g is initially zero, but as pressure decreases, water begins to exsolve out of the melt and forms bubbles. The absolute volume of exolved gas (V) can be calculated through re-arranging the ideal gas law (DR6). This absolute volume of gas is then used to calculate the volume fraction of the bulk magma constituted by a gas phase.

The bulk magma viscosity (η) is determined by first calculating the viscosity of the pure melt phase (η_m) using Hess and Dingwell, 1996. When the effect of crystals within the melt is considered, the viscosity of the melt and crystal mixture (η_{mc}) increases, and is represented by the Einstein-Roscoe equation:

$$\eta_{\mathrm{mc}} = \eta_{\mathrm{m}} \left(1 - \frac{\chi_{\mathrm{c}}}{\chi_{\mathrm{c}}{}^{\mathrm{max}}}\right)^{-2.5} \ , \quad \text{(DR7)}$$

where χ_c^{max} is the volume fraction of crystals at which the maximum packing is achieved (Marsh, 1981). Additionally, the presence of bubbles (generated by the volume fraction of gas) also effects η. If the bubbles within the magma remain un-deformed they act to increase η but if they are elongated in the direction of flow they act to decrease η (Llewellin and Manga, 2005). Whether a bubble is un-deformed or deformed can be calculated through the capillary number (Ca):

$$Ca = \frac{\eta_{\mathrm{m}} r \varepsilon}{\Gamma} \ , \quad \text{(DR8)}$$

where r is the unreformed bubble radius, Γ, the bubble surface tension, and ε, a function of the strain rate of the magma flow. If $Ca > 1$, then the bubbles can be considered deformed. Previous authors have calculated Ca as a function of only shear strain rate (Colllier and Neuberg, 2006) or of both shear strain rate and the rate of change of shear strain rate (Llewellin and Manga, 2005). Here we introduce the concept of not only shearing, but also stretching the bubbles and calculate Ca as a function of shear strain rate, elongational strain rate and the rate of change of strain rate. Depending on the value of Ca, η is calculated using the suggested 'minimum variation' of Llewellin and Manga, 2005:

If $Ca < 1: \eta = \eta_{mc}(1-\chi_g)^{-1}$: If $Ca > 1: \eta = \eta_{mc}(1-\chi_g)^{5/3}$ (DR9)

By assuming the homogeneous nucleation of a number of bubbles in a unit volume of melt, which is determined from the initial bubble number density (b_{ni}) (Hurwits and Navon, 1994), the bubble radius (Lensky et al., 2002) is given by:

$$r = \left(\frac{S_0{}^3 \rho_m (C_0 - C_m)}{\rho_g}\right)^{1/3} , \quad \text{(DR10)}$$

where C_0 and C_m are the initial and remaining amount of water dissolved in the melt respectively and S_0 is the initial size of the melt shell from which each bubble grows. S_0 is related to the instantaneous bubble number density (b_n) through the expression:

$$S_0{}^3 = \frac{3}{4\pi b_n} \quad \text{(DR11)}$$

b_n is used rather than b_{ni} as since homogeneous nucleation is assumed, the bubble number density must remain constant with respect to the volume of the melt fraction of the bulk magma. b_n is given by:

$$b_n = \frac{b_{ni}}{\chi_m}\left(\chi_m - (1-\chi_g)\right) . \quad \text{(DR12)}$$

Brittle Failure of Melt

The failure of magma in a shear sense according to the brittle failure criterion (1) is proposed as the source mechanism of the seismicity discussed in this paper. This criterion holds true under the assumption that during un-relaxed deformation the accumulation of shear stress in the magma obeys the Maxwell model:

$$\sigma_s = \frac{\eta_s}{G}\frac{\partial \sigma_s}{\partial t} = \dot{\varepsilon}\eta , \quad \text{(DR13)}$$

where σ_s is the shear stress and G the elastic modulus.

Boundary Conditions

Flow within the system is driven by a pressure gradient defined by boundary conditions at the top and bottom of the conduit. The top boundary is set to atmospheric pressure, while the bottom boundary is set to lithostatic pressure, assuming a homogeneous country rock density of 2600 kg m^{-3}. Both the top and bottom pressure conditions are held constant throughout the model run. Initial boundary conditions along the length of the conduit are defined as no slip. Upon a stationary solution being returned, the results are assessed, and along regions of the conduit wall where the brittle failure criterion was exceeded, the boundary conditions are changed to a tangential slip velocity ($\Delta \boldsymbol{u}$) defined by:

$$\Delta \boldsymbol{u} = \frac{1}{\beta} \tau_s, ,$$ (DR14)

where τ_s is the tangential shear stress to the conduit wall and the coefficient β is a function of the slip length (L_s) which is defined as:

$$\beta = \frac{\eta}{L_s},$$ (DR15)

the model is then re-run to observe the effect of changing boundary conditions.

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