Supplementary Information

Multiphase Numerical Calculations

We used an Eulerian-Eulerian multiphase approach to compute crystallization and silicate melt extraction in a magma chamber that is progressively cooling. Each continuum (magma or crystal phase) is represented by conservation equations for the mass, momentum, thermal energy and a compositional proxy (SiO_2). The set of equations are coupled through equal and opposite drag terms, terms for the transfer of thermal energy, and terms for the transfer of mass during phase change:

$$\sum_k \phi_k = 1 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}}\right) = R_k \tag{2}$$

$$\frac{\partial(\phi_k \rho_k \mathbf{u_{k,i}})}{\partial t} + \frac{\partial(\phi_k \rho_k \mathbf{u_{k,i}} \mathbf{u_{k,j}})}{\partial \mathbf{x_i}} = -\phi_k \frac{\partial P}{\partial \mathbf{x_i}} \delta_{ij} + \frac{\partial}{\partial \mathbf{x_i}}\left[\tau_{\mathbf{ij}}\right] + \mathbf{D}_i + \rho_k \phi_k \boldsymbol{g}_2 \delta_{\mathbf{i2}} + R_k \mathbf{u_{k,i}} \tag{3}$$

$$\phi_k \rho_k c_k \left[\frac{\partial \mathrm{T_k}}{\partial \mathrm{t}} + \mathbf{u_i} \frac{\partial \mathrm{T_k}}{\partial \mathbf{x_i}}\right] = \delta_{\mathrm{km}} \frac{\partial \mathrm{q_k}}{\partial \mathbf{x_i}} + \pi k_m \mathrm{d}\ \mathbf{Nu}\left(\mathrm{T}_m - \mathrm{T_c}\right) + \phi_k R_k L \tag{4}$$

$$\frac{\partial}{\partial t}\left(\phi_k \rho_k C_{SiO_2}\right) + \frac{\partial}{\partial \mathbf{x_i}}\left(\phi_k \rho_k \mathbf{u_{k,i}} C_{SiO_2}\right) = \beta_{(f)} \tag{5}$$

The *k* subscript refers to specific phases (magma (*m*) or crystal (*c*)) and each phase is described by separate sets of equations. The subscripts *i* and *j* refer to spatial directions that can take the values of 1 and 2 for the two orthogonal directions in the two-dimensional simulations. Here we use the convention that 2 is in the z or vertical

direction. T refers to temperature, q to heat flux, d to crystal diameter, k to thermal conductivity, c to heat capacity, ρ to density, L to latent heat, $\mathbf{u}$ to velocity vectors, τ_{ij} to the stress tensor, and ϕ to volume fraction. The first equation, (1), specifies that at any one location the sum of volume fraction of magma and crystals must equal 1. Equation (2) is the conservation of mass, equation (3) gives the momentum balance, equation (4) is the conservation equation for thermal energy, and equation (5) is the conservation of the chemical proxy (SiO_2) content of the magma. The term $\beta_{(f)}$ in equation 5 is the rate of change of composition due to phase change as parameterized by the changing melt fraction as parameterized in MELTS calculations (Ghiorso and Sack, 1995). Hence this term is based on heat transfer rates, kinetic rates, and the phase equilibria predicted by the self-consistent thermodynamics approach incorporated in MELTS.

The R_k terms are the mass exchange rates associated with crystallization. During crystallization the R_k value for the magma phase would be positive and have an equal, but opposite complementary term in the crystal phase equation. The R_k terms also appear in the conservation of momentum equations (Eq. 3). This term takes on the form:

$$R_k = \frac{\left(f^t - f^{t-1}\right)}{\left|f^t - f^{t-1}\right|}\lambda_{xl} \tag{6}$$

Where f^t and f^{t-1} are the melt volume fraction as a function of temperature at two times separated by a Δt time span. For these calculations we assume a linear relationship between melt fraction and temperature when magmatic temperature is between the solidus and liquidus of the magma. The rate of crystallization, λ_{xl}, is ultimately governed by the crystallization kinetics. Here we use a rate of 10^{-4} kg/m^{3}s. This is equivalent to a growth rate of $\sim 1.0 \times 10^{-9}$ mm/s assuming initial crystal sizes of 100 microns and nucleation number density of 10^{12} nucleation sites/m^3. Growth rates for multicomponent compositions have been reported in the range of 10^{-6} to 10^{-11} mm/s (Hammer, 2008).

The heat transfer between the magma and crystals is given by the second term in equation 4 and is modified by the Nusselt number, **Nu**. We use the Ranz-Marshall correlation to prescribe the Nusselt number based upon the particle Reynolds number and Prandtl number (Rowe et al., 1965),

$$\mathbf{Nu} = 2 + \frac{4}{5}\mathbf{Re_p}^{1/2}\mathbf{Pr}^{1/3} \tag{7}$$

where

$$\mathbf{Re_p} = \frac{d\rho_m \left|\mathbf{u_{m,i}} - \mathbf{u_{c,i}}\right|}{\mu_m}, \tag{8}$$

and

$$\mathbf{Pr} = \frac{\mu_m c_m}{k_m}. \tag{9}$$

The drag between magma and crystals, **D**, is dependent on the volume fraction of crystals. We assume that above the close packing volume fraction of crystals (here assumed to be 0.64), the single particle drag factor is based on particle Reynolds number using the Schiller and Naumann correlation, and the total drag factor of a collection of particles is given by the correlation of Di Felice to account for hindered settling at higher particle volume fraction (Di Felice, 1994).

$\phi_c < 0.64$:

$$\mathbf{D_i} = \frac{f}{t_c}(\mathbf{u_{m,i}} - \mathbf{u_{c,i}}) \tag{10}$$

Here f is the total drag correlation,

$$f = f_0 \phi_m^{-\beta}. \tag{11}$$

The single particle drag correlation is given by (Crowe et al., 1998):

$$f_0 = 1 + .15\mathbf{Re_p}^{0.687}, \tag{12}$$

which is appropriate for the relatively limited particle Reynolds numbers encountered in magmatic flow. The exponent, β, is given as

$$\beta = 3.7 - 0.65 \exp\left[\frac{-\left[1.5 - \log(\mathbf{Re_p})\right]^2}{2.0}\right]. \tag{13}$$

The crystal's response time, t_c, is

$$t_c = \frac{\rho_c d_c^{\,2}}{18\mu_m}. \tag{14}$$

When the volume fraction of crystals exceeds the close packing limit, the drag force is given

$$\mathbf{D_i} = \frac{\mu_m}{K}(\mathbf{u_{m,i}} - \mathbf{u_{c,i}}), \tag{15}$$

where the permeability, K, is given by the Blake-Karmen-Kozeny relationship

$$K = \frac{\phi_m^{\,3} d_c^{\,2}}{8M(1-\phi_m^{\,2})}. \tag{16}$$

Here M is a constant with a value of 50. With the permeability relationship and Darcy's law formulation for the drag force, the solved multiphase equations correspond to the compaction equations with the exception that the inertial terms are retained to be able to smoothly transition from a melt dominated to a crystal dominated system(McKenzie, 1985).

The stress tensors, $\tau_{k,ij}$, are given as

$$\tau_{k,ij} = 2\mu_k S_{k,ij}, \tag{17}$$

where

$$\mathbf{S_{k,ij}} = \frac{1}{2}\left[\frac{\partial \mathbf{u_{k,i}}}{\partial x_j} + \frac{\partial \mathbf{u_{k,i}}}{\partial x_i}\right] - \frac{1}{3}\frac{\partial \mathbf{u_{k,i}}}{\partial x_i}\delta_{ij} \tag{18}$$

The evolving viscosity and thermal properties for the magma phase were computed using MELTS. The initial proxy compositions are given in Supplemental Table 1, for basaltic, andesitic and dacitic magmas. While variations in initial composition, and rheology, are certainly possible, these calculations demonstrate that these end-member compositions have similar behavior in terms of the cumulative probability of melt extraction, although the rapidity of melt extraction varies. We assumed a constant solid matrix viscosity of 10^{14} Pa s after crystal lock-up (Caricchi et al., 2007; Rushmer, 1995).

We solved these equations using a modified version of the multiphase flow with interphase exchanges (MFIX) developed by the U.S. Department of Energy(Syamlal et al., 1993) with modifications for magmatic and compaction driven flow (Dufek and Bergantz, 2007; Dufek and Bergantz, 2005; Ruprecht et al., 2008). We have validated this approach through comparison of experimental and direct numerical simulations of gravity currents (Dufek and Bergantz, 2007), and through comparison with rates of sedimentation of silicon particle in molten aluminum (Hanumanth et al., 1992). All

simulations are two-dimensional and we used a constant grid resolution 0.5 m by 0.5 m. We used a flux-limiting approach (Zijlema and Wesseling, 1998) to maintain second order accuracy when solving the conservation equations using the finite volume approach.

Modeled Conditions

We focused primarily on low aspect ratio geometries (height/length). Sills with low aspect ratio appear to be favored in many tectonic settings, and gravitational relaxation tends to create rheological and density traps that are horizontal in nature. In the shallow crust, most caldera forming eruptions also have very limited aspect ratios. We focused on magma chambers with an aspect ratio of 0.25. Calculations for smaller aspect ratios behaved very similar to the 0.25 aspect ratio simulations in terms of the probability of melt extraction. We considered magma intrusion heights of 10 m and 100 m.

In the multiphase calculations three compositional proxies were considered for basalt, andesite and dacite magmas, respectively. To compute the thermodynamic properties of these proxies we used the specific compositions given in Table 1 and used the MELTS thermodynamic software to compute the heat capacity, density, and viscosity of these melts as a function of non-dimensional temperature. In all calculations we assumed 2 wt. % water, although we examined other scenarios, and the probability of melt extraction was relatively insensitive to variation in water and composition (although the timescale of melt extraction was longer for less hydrous, more viscous magmas).

The cumulative melt extraction is computed at every time-step and is defined as:

$$V_{extract} = \sum \phi_m \left(u_{m,2} - u_{c,2} \right) dx_1 dx_2 dt \,, \tag{19}$$

where $V_{extract}$ is the cumulative volume of melt separating from the crystals. The '2' in the subscript refers to the vertical direction. Extracted volumes are also computed for specific crystal fractions. These crystal fraction-specific volumes are also computed every time-

step and are binned for every 0.02 volume fraction of crystals. The probability of extraction as a function of crystal content is computed at the conclusion of the simulations by taking the ratio of the crystal fraction specific volumes with the total extracted melt volume, and multiplying by 100 for a percentage.

The two-dimensional simulations were linked to broader-scale, one-dimensional extension modeling the temperature in the lithosphere (here assumed to be 100 km). The initial condition assumed a steady-state thermal profile with a surface heat flux of 68 mW/m^2 using the approach of Chapman and Furlong (1992). Heat production from radioactive elements was assume to be 0.94 mW/m^3 at the surface with an exponential decrease with depth with a characteristic length scale of 15 km. Obviously other thermal profiles are possible, but the connection to the one-dimensional model enables us to test the melt extraction probability when the intrusion is at different degrees of thermal disequilibrium from the crust, and also allows the boundary of the intrusion to change in response to conduction of heat to the surrounding crust rather than giving ad hoc temperature or heat flux boundary conditions for the magma intrusion.

Table DR1: Modeled Compositions

Major Oxide Composition	**Basalt**	**Andesite**	**Dacite**
SiO_2	49.61	61.19	68.0
TiO_2	1.0	1.17	0.45
Al_2O_3	17.41	17.22	15.63
Fe_2O_3	11.31	6.32	3.70
MnO	.18	.01	.07
MgO	6.11	1.23	.93
CaO	10.72	4.43	2.98
Na_2O	2.74	4.18	3.78
K_2O	1.23	3.65	4.10
P_2O_5	.22	.61	.19

References:

Caricchi, L., Burlini, L., Ulmer, P., Gerya, T., Vassalli, M., and Papale, P., 2007, Non-Newtonian rheology of crystal-bearing magmas and implications for magma ascent dynamics: Earth and Planetary Science Letters, v. 264, p. 402-419.

Chapman, D.S., and Furlong, K.P., 1992, Thermal state of the continental lower crust, *in* Fountain, D.M., Arculus, R. and Kay, R.W., ed., Continental Lower Crust: Amsterdam, Elsevier.

Crowe, C.T., Sommerfeld, M., and Tsuji, Y., 1998, Multiphase Flows with Droplets and Particles: New York, CRC Press.

Di Felice, R., 1994, The voidage function for fluid-particle interaction systems: International Journal of Multiphase Flow, v. 20, p. 153-159.

Dufek, J., and Bergantz, G.W., 2007, The suspended-load and bed-load transport of particle laden gravity currents: Insight from pyroclastic flows that traverse water: Journal of Theoretical and Computational Fluid Dynamics, v. 21, p. 119-145.

Dufek, J.D., and Bergantz, G.W., 2005, Transient two-dimensional dynamics in the upper conduit of a rhyolitic eruption: A comparison of the closure models for the granular stress: Journal of Volcanology and Geothermal Research, v. 143, p. 113-132.

Ghiorso, M.S., and Sack, R.O., 1995, Chemical mass transfer in magmatic processes IV. A revised and internally consistent thermodynamic model for the interpolation and extrapolation of liquid-solid equilibria in magmatic systems at elevated temperatures and pressures: Contributions to Mineralogy and Petrology, v. 119, p. 197-212.

Hammer, J.E., 2008, Experimental studies of the kinetics and energetics of magma crystallization, *in* Putrika, K.D., and Tepley, F.J., eds., Minerals, inclusions and volcanic processes, Volume 69, Mineralogical Society of America, p. 9-59.

Hanumanth, G.S., Irons, G.A., and Lafreniere, S., 1992, Particle sedimentation during processing of liquid metal-matrix composites: Metallurgical Transactions B, v. 23, p. 753-763.

McKenzie, D., 1985, The extraction of magma from crust and mantle: Earth and Planetary Science Letters, v. 74, p. 81-91.

Rowe, P.N., Claxton, K.T., and Lewis, J.B., 1965, Heat and mass transfer from a single sphere in an extensive flowing fluid: Transactions of Institution of Chemical Engineers, v. 43, p. T14.

Ruprecht, P., Bergantz, G.W., and Dufek, J., 2008, Modeling of gas-driven magmatic over-turn:Tracking of phenocryst dispersal and gathering during magma mixing: GEOCHEMISTRY GEOPHYSICS GEOSYSTEMS, v. 9, p. doi:10.1029/2008GC002022.

Rushmer, T., 1995, An experimental deformation of partially molten amphibolite: Application to low-melt fraction segregation: Journal of Geophysical Research, v. 100, p. 15,681-15,695.

Syamlal, M., Rogers, W., and O'Brien, T.J., 1993, MFIX Documentation:Theory Guide: Morgantown, WV, U.S. Department of Energy, 1-49 p.

Zijlema, M., and Wesseling, P., 1998, Higher-order flux-limiting schemes for the finite volume computation of incompressible flow: International Journal of Computational Fluid Dynamics, v. 9, p. 89-109.