

Strictly speaking, Equation 6 in Kump et al. (2005) is inappropriate for characterizing regional euxinia created by intense local upwelling, as the problem becomes 2- or 3-dimensional. However, the following analysis demonstrates that the inclusion of lateral transport is unimportant for this problem.

To include the non-local sink for H₂S requires an additional term in equation 6 that accounts for horizontal eddy diffusion (D_h , $\text{m}^2 \cdot \text{yr}^{-1}$) (cf. the treatment of worm burrow irrigation by Boudreau, 1997). D_h is scale-dependent (Okubo, 1971); here we have chosen $L=1000$ km, which gives $D_h \approx 3 \times 10^{10} \text{ m}^2 \cdot \text{yr}$. We can then express

horizontal diffusion in terms of a horizontal piston velocity $\left(\frac{D_h}{L}\right)$. If the flux is radially symmetric, then the

horizontal flux of H₂S from the upwelling region is through a cross-sectional area of $2\pi LZ$, where Z is the surface layer thickness, and the vertical upwelling flux of H₂S from below and the in-mixing flux of O₂ from above are through a cross-sectional area πL^2 . Thus, we multiply the maximum horizontal diffusive flux

$\left(\frac{D_h}{L} \cdot \rho_{\text{oce}} \cdot [\text{H}_2\text{S}]_{\text{deep}}\right)$, when $[\text{H}_2\text{S}]_{\text{surf}} = [\text{H}_2\text{S}]_{\text{deep}}$, by the ratio of these areas $\left(\frac{Z}{L}\right)$ and insert this flux into

equation 6. This leads to the following revised steady-state expression for the critical ratio above which the surface layer becomes euxinic given the conditions applied to equation 6:

$$\begin{aligned} \left(\frac{[\text{H}_2\text{S}]_{\text{deep}}}{P_{\text{O}_2, \text{atm}}}\right)_{\text{crit}} &= \frac{k \cdot K_H}{2 \cdot \left(u - 2 \cdot \frac{D_h}{L} \cdot \frac{Z}{L}\right)} = \\ &= \frac{1000(\text{m} \cdot \text{yr}^{-1}) \cdot 10^{-3} \text{ mol} \cdot \text{kg}^{-1} \cdot \text{bar}^{-1}}{2 \cdot \left(100 \text{ m} \cdot \text{yr}^{-1} - 2 \cdot \frac{3 \times 10^{10} \text{ m}^2 \cdot \text{yr}^{-1}}{10^6 \text{ m}} \cdot \frac{10^2 \text{ m}}{10^6 \text{ m}}\right)} = 0.005 \frac{\text{mol}}{\text{kg} \cdot \text{bar}} \end{aligned}$$

For an atmospheric $p\text{O}_2$ of 0.21 bar, the critical $[\text{H}_2\text{S}]_{\text{deep}}$ is $1 \text{ mmol} \cdot \text{kg}^{-1}$, virtually identical to that obtained by using equation 6 with the appropriate upwelling velocity. In other words, lateral transport is not an important consideration for this problem.

References

- Boudreau, B. P., 1997, Diagenetic Models and their Implementation: Berlin, Springer, 414 p.
 Okubo, A., 1971, Oceanic diffusion diagrams: Deep-Sea Research, v. 18, p. 789-802.