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Viscous Fingering During Replenishment Of Felsic Magma Chambers By Repeated Inputs Of Mafic Magmas: Field Evidence And Fluid-Mechanics Experiments

by

Perugini D. & Poli G.

1. ADDITIONAL INFORMATION ABOUT FRACTAL ANALYSIS

A fractal is an 'object' whose shape is irregular and/or fragmented at all scales. Mathematically, a fractal is defined as a set for which the Hausdorff-Besicovich dimension (or fractal dimension D) strictly exceeds the Euclidean dimension, D_E, which is always an integer (see Mandelbrot, 1982 for details). For instance the value of D for a fractal curve (i.e. an extremely convoluted curve with many gulfs at all scales) lying on a 2D plane, lies between the value of D_E for the straight line, which is unity, and the dimension of the plane, that is two. A basic property of fractals is their 'scaling' behaviour (or self-similarity). Self-similarity implies that every part of the object is a reduced version of the whole. Two classes of fractals exist: 'mathematical' and 'natural' fractals. In mathematical fractals, their self-similarity property is assumed to hold good for the entire spectrum of time or space scales. In natural fractals, which are encountered in Nature and which are the concern of our study, small- and large-scale constraints usually confine the self-similarity property to a finite range of scales. Fractal natural structures are quantified by measuring their fractal dimension (D), a measure of their degree of complexity. In details, the fractal dimension increases as the complexity of the structure increases. There are several methods to measure fractal dimension of natural structures and most of them are implemented into computer codes that allow fast and accurate fractal dimension estimates (e.g. Perugini, 2002).

One of the most used technique to measure fractals dimension of structures on digital images, such as the mafic/felsic magma interfaces considered in this study, is known as the box-counting method. Before applying this technique, grayscale pictures (Fig. DR1A) need to be converted to binary black and white images. This is done by thresholding grey scale images, to produce images in which mafic and felsic magmas are replaced by black and white, respectively (Fig. DR1B). Then, the interface between the two magmas is detected by image analysis by tracing the contact between the black and white pixels in the image (Fig. DR1C) and the box-counting technique can be performed.

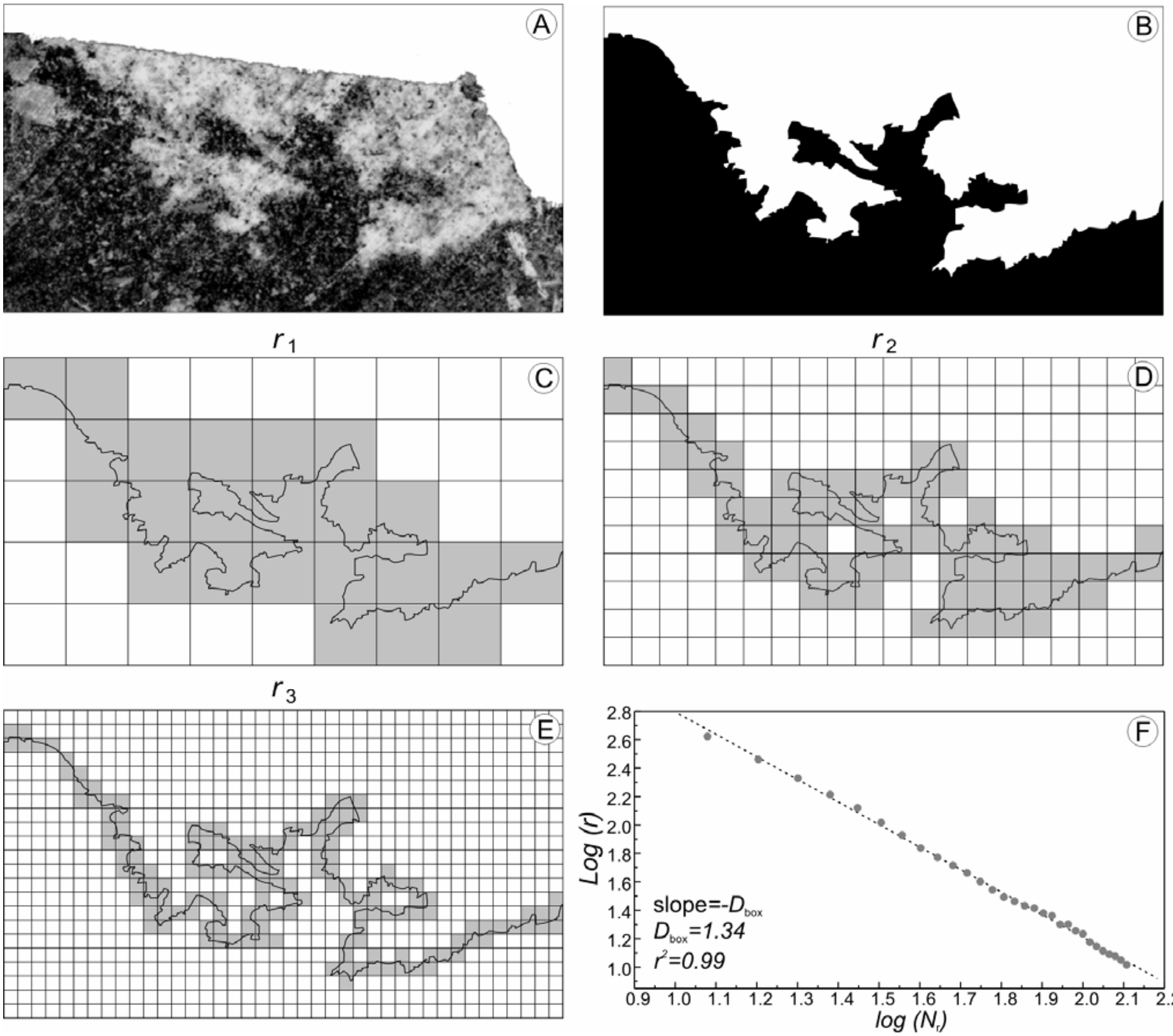


Figure DR1. Procedure used to measure fractal dimension (D_{box}) of mafic felsic interfaces. A) original grayscale image; B) image in A reduced to a binary black and white image by applying a threshold level; C-E) box-counting method: a square mesh of various sizes (r_1, r_2, etc.) is laid over image, and number of boxes (N_r) containing black pixels associated with interface between fluids is counted; F) fractal dimension (D_{box}), calculated by linear interpolation of log(r) vs. log(N_r) graph; slope of linear interpolation is equal to -D_{box}. The fractal nature of the structures is demonstrated by the excellent linear fitting of data (r^2 always larger than 0.95; e.g. Bruno et al., 1994). Note the good fractal scaling over more than one order of magnitude.

A square mesh of size (r) is laid over the image and the number of boxes (N_r) containing the black pixels associated with the interface between magmas is counted (Fig. DR1C-E). Mandelbrot (1982) showed that, for fractal patterns, the following relationship is satisfied:

$$N_r = r^{-D_{box}} \quad (1)$$

Using logarithms, (1) may also be written as follows:

$$\log(N_{\mathrm{r}}) = -D_{\mathrm{box}} \cdot log\ (r) \qquad (2)$$

This relationship shows that in order to classify a structure as a fractal data must lay on a straight line in the log-log plot where the fractal dimension (D_{box}) is calculated as the slope resulting from the linear interpolation of the log(r) vs. log(N_{r}) graph.

Fig. DR1F shows an example of log-log plot for an interface separating the mafic and felsic magma and shows that data points follow a linear distribution as evidenced by the very high value of r^2 (=0.999) of the linear fitting, demonstrating that interfaces are fractals. In average 30 different meshes with different size (r) are utilized for measuring D_{box} of each structure and this allows us to appreciate a very good fractal scaling over more than one order of magnitude.

Regarding the goodness of the linear fitting, it is to note that r^2 values are higher than 0.95 (Bruno et al., 1994) for all analyzed interfaces and this demonstrates that mafic/felsic interfaces considered in this study are to be considered as natural fractals. All fractal dimension measurements have been performed by the software MorphoUt 1.0 (Perugini, 2002). This software has been extensively tested on structures with known fractal dimension and results are found to be very accurate [see Perugini (2002) for details].

2. ERRORS ON D_{box} OF NATURAL STRUCTURES

The uncertainty of D_{box} due to the reduction of the original grayscale images to black and white images has been checked by performing several measurements of fractal dimension on binary images of the same structure obtained at different threshold levels. The graph of Fig. DR2 shows the variation of fractal dimension with the threshold level (in grey values). To understand the graph we must recall that the mafic magma is dark colored (lower grey values) whereas the felsic magma tends to be clear colored (higher grey values; Fig. DR1A and Fig. 1 in the main text). When a threshold level is applied to digital images all grey values below this threshold will be reduced to the black color, whereas all grey values above the threshold will be reduced to the white color (Fig. DR1B). The graph of Fig. DR2 indicates that for a large range of threshold levels (from 45 to 180 grey value), the fractal dimension of the interface between magmas shows very little variation leading to fractal dimension estimates with an error better than 0.5%. This happens because the color contrast between the mafic and felsic magmas is very strong and this allows us to separate very well the interface between magmas for a large range of thresholds. For grey values lower than 45 and higher than 180 fractal dimension strongly fluctuates. This is due to the fact that below 45 and above 180 the images suffer the interference of grey values that do not constitute the mafic/felsic interface and, therefore, the interface cannot be traced precisely. All values of fractal dimension presented in our study are measured in the range of threshold values where *D* shows very little variations (errors better than 0.5%).

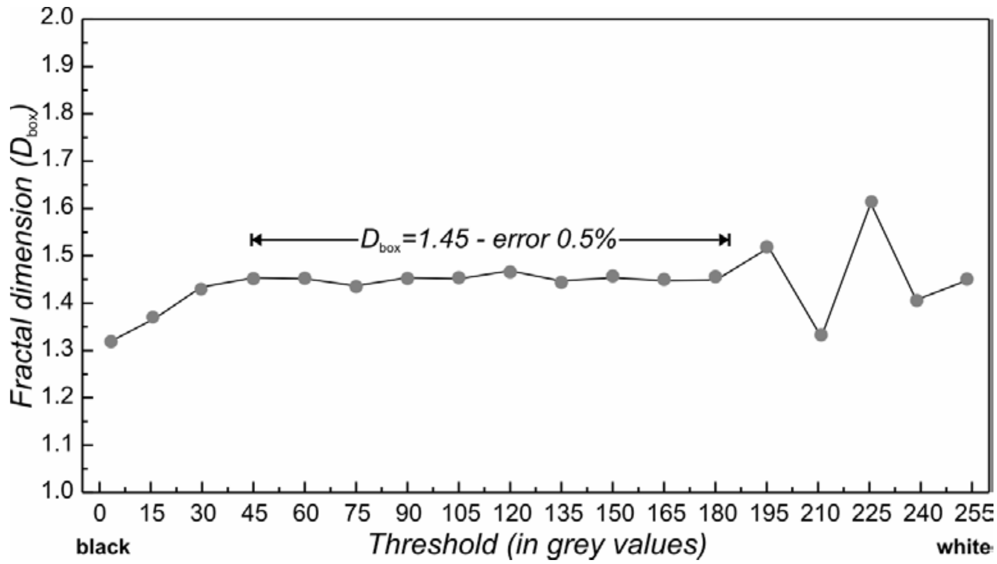


Figure DR2. Variation of fractal dimension as a function of the threshold level.

3. ERRORS ON D_{box} OF SIMULATED STRUCTURES

Tests have been performed to check the reproducibility of D_{box} by performing five experiments at the same viscosity ratio; results show that D_{box} can be estimated with an error better than 0.05%.

Tests have been also performed to check the dependence of D_{box} values on the injection rate of the low-viscosity fluid. Experiments have been performed with injection rates from 0.5 to 2.5 mL/s; results indicate that, in the investigated range of injection rates, D_{box} can be estimated with an error better than 1.0%.

4. COMPARISON BETWEEN NATURAL AND EXPERIMENTAL STRUCTURES

A further aspect that is worth discussing is the comparison between fractal dimensions of natural and experimental structures. In fact, while natural structures are sections of *3D* morphologies, experimental structures are the result of *2D* experiments.

To our knowledge there are not viscous fingering experiments in *3D* and up to now the *2D* experiments are the only method to study this complex phenomenon. It is important to note that *2D* experiments are commonly conceived as proxies for *2D* sections of *3D* structures (e.g.Vicseck, 1992) and, as such, they are utilised in our study. However one may ask how the fractal dimension of natural structures measured on planar sections is related to the fractal dimension of the whole *3D* structure. In particular, problems may arise if the fractal dimension from different sections of the same 3D structure are different. Since 3D viscous fingering experiments are not available at present to solve this problem, we can discuss this issue only theoretically by taking into account the additive properties of codimensions of intersections (Mandelbrot, 1982). Specifically, from a topological point of view, if S_1 and S_2 are two independent sets embedded in a space of dimension d, and if $codimension(S_1)+codimension(S_2)<d$, the codimension of $S_1 \cap S_2$ is the sum of the codimensions of S_1 and S_2. For a fractal set F embedded in a 3D space and intersected by a plane, the above statements implies that the dimension of the intersected set is one less that the dimension of F. In practice, if, for instance, the fractal dimension of a 3D structure (D_s) is 2.45, the fractal dimension of all possible intersections (D_I) will be D_I =2.45-1=1.45. This demonstrates that different sections of the same 3D structure have the same fractal dimension and that a single section fully characterises the studied 3D structure. Therefore, from a theoretical point of view, the comparison of fractal dimensions measured on 2D natural structures and structures resulting from 2D experiments is justified.

It is important to note that although these considerations have not been proven for real viscous fingering structures, they have been experimentally proven for turbulent jets (e.g. Sreenivasan et al., 1989), indicating their applicability to real world problems and supporting our approach to compare experimental and natural viscous fingering fractal dimensions.

REFERENCES CITED

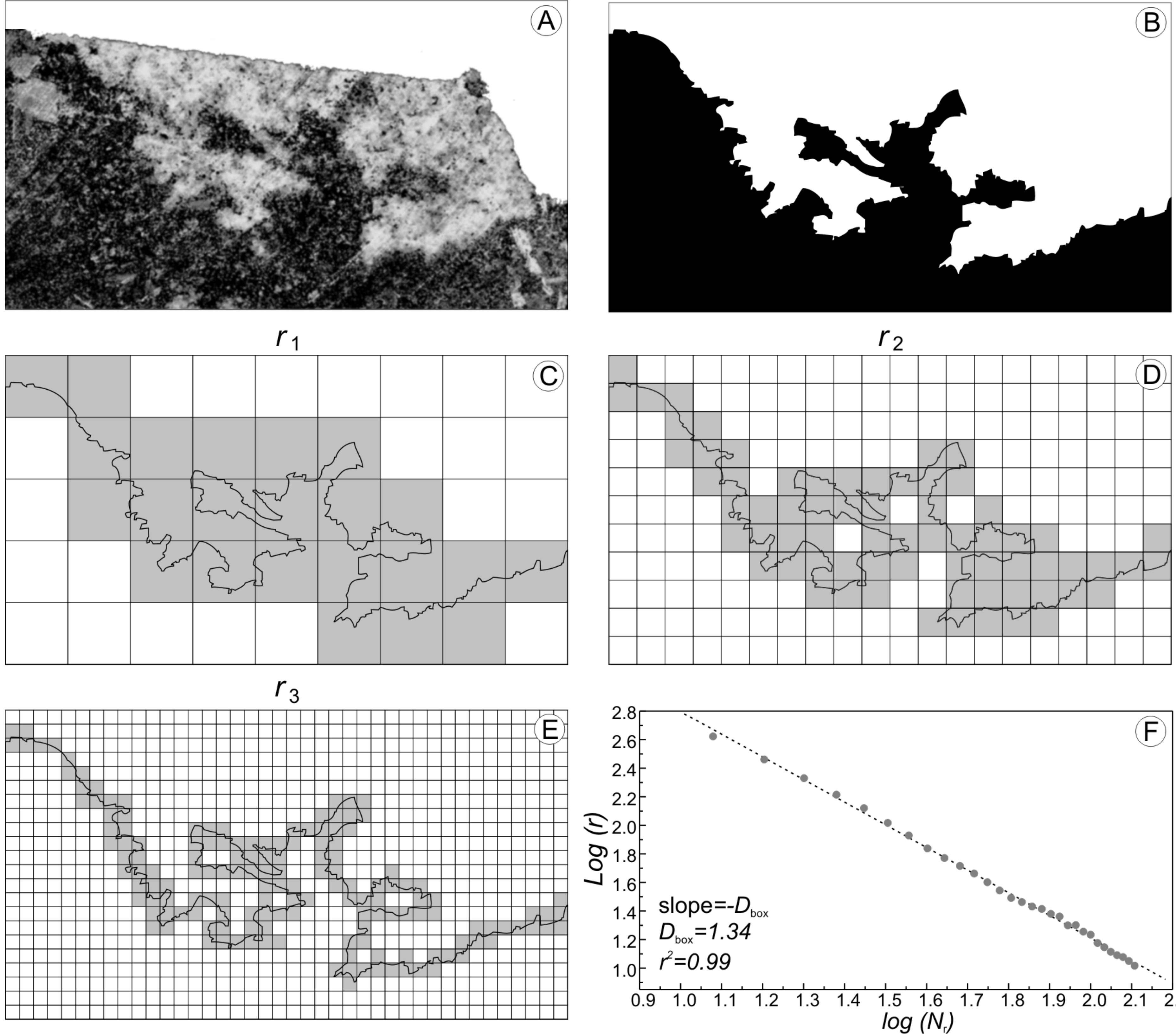
Mandelbrot, B.B., 1982, The fractal geometry of nature, W. H. Freeman, New York, p. 468.

Perugini, D., 2002, MorphoUt 1.0: Utilities for Closed Shape Morphometry: Computers and Geosciences, v. 28, p. 73-79.

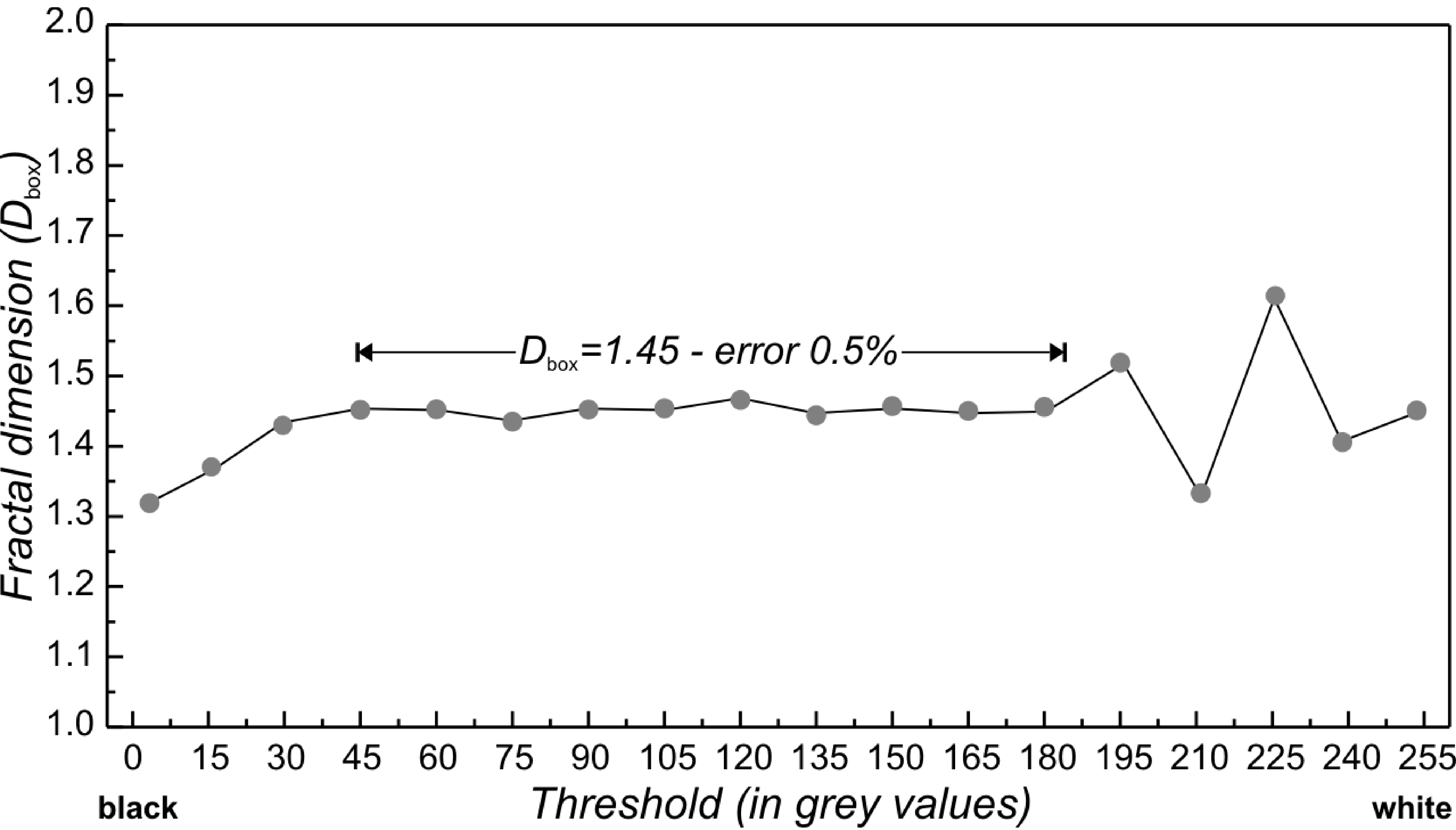
Bruno, B.C., Taylor, G.J., Rowland, S.K., and Baloga, S.M., 1994, Quantifying the effect of rheology on lava-flow margins using fractal geometry: Bulletin of Volcanology, v. 56, p. 193-206

Sreenivasan, K.R., Prasad, R.R., Mendeveau C., and Ramshankar, R., 1989, The fractal geometry of interfaces and the multifractal distribution of dissipation in fully turbulent flows. In: Fractals in geophysics (ed. C. H. Scholtz e B. B. Mandelbrot). Birkhauser Verlag, Basilea, p. 313.

Vicsek, T., 1992, Fractal Growth Phenomena, 2nd ed. World Scientific, Singapore, p. 488.



A
B
r_1
C
r_2
D
r_3
E
F
slope=-D_{box}
D_{box}=1.34
r^2=0.99
Log (r)
log (N_r)
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2


Fractal dimension (D_{box})
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
D_{box}=1.45 - error 0.5%
0 15 30 45 60 75 90 105 120 135 150 165 180 195 210 225 240 255
black
Threshold (in grey values)
white