

Additional information on methods:

The original data are contours of subglacial topography. Contours were digitized then rasterized onto a uniform grid using Generic Mapping Tools (GMT) (Smith and Wessel, 1990). Co-ordinates of grid are : x = 0, y = 0: 85 °S, 120 °W; x = 0, y = 300 km: 87.57 °S, 113 °W; x = 1180 km, y = 300 km: 80.17 °S, 147.5 °E; x = 1180 km, y = 0: 79.24 °S, 162 °E

The surface fitted to the summits in fig. 3B is done with the surface program within Generic Mapping Tools (GMT) (Smith and Wessel, 1990).

Isostatic response function

The elastic flexure equation (Watts, 2001) is solved for a 3D elastic sheet using finite difference methods (Stern et al., 1992) . We use the most general partial differential equation describing deformation of a thin elastic plate (Timoshenko & Woinowsky-Krieger , 1959):

$$\nabla^2(D\nabla^2W) - (1-\nu)\left(\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 W}{\partial y^2} - \frac{2\partial^2 D}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y} + \frac{\partial^2 D}{\partial y^2}\frac{\partial^2 W}{\partial x^2}\right) - F_x\frac{\partial^2 W}{\partial x^2} - F_y\frac{\partial^2 W}{\partial y^2} - F_{xy}\frac{\partial^2 W}{\partial x\partial y} + \gamma W = P(x,y)$$

where : (x,y) = Cartesian coordinates

∇^2 = Laplace operator

D (x,y) = flexural rigidity = $E T_e^3 / (12(1-\nu^2))$

E = Young's modulus

ν = Poisson's ratio

T_e = plate thickness

W (x,y) = vertical displacement

γ = buoyancy induced stress per unit of vertical displacement (or restoring force)

P(x,y) – vertically acting load

F_x = in-plane force in x-direction

F_y = in-plane force in y-direction

F_{xy} = in-plane shear force

A uniform 5 x 5 km grid is superimposed on the sheet and values of T_e , load (in MPa) and restoring force (in g^* density contrast(kg/m^3)) is allocated to each grid location. One of the four edges of the elastic plate can be specified as being a free-edge in the sense that no shear stresses can be transmitted over it and the displacements are not required to be zero at the edge. In this study the missing rock due to incision is the load (assumed density of rock = 2700 kg/m^3) and the restoring force is then $g \Delta\rho$, where $\Delta\rho = \rho_{\text{mantle}} - \rho_{\text{air}} = \rho_{\text{mantle}} = 3300 \text{ kg/m}^3$, as in isostatic rebound mantle is effectively displacing air.

Geological applications of this equation generally make one or more of the simplifying assumptions: that the flexural rigidity D is spatially uniform, that in-plane stresses are negligible and the driving load and deformation vary in one direction only. In this study we make all these assumptions apart from allowing D (flexural rigidity), restoring force (γ) and load (P) to vary in both x and y directions. Our 3D code was bench-marked for simple shaped loads for which standard analytical solutions exist. These benchmark tests are described in Stern et al. (1992).

Relationship between width of valley, flexural parameter and rebound:

The wavelength of flexure is controlled by the flexural parameter (α) = $4 D / [(\Delta\rho)g]$, where D is the flexural rigidity and $\Delta\rho$ is the density contrast between the mantle and infilling material above the plate (air in the case of rebound) that is displaced by the flexure. If L is the wavelength of erosion, then the dimensionless parameter L/α is what controls the degree of isostatic compensation (Walcott, 1970). For example, with a low $T_e = 5 \text{ km}$, $\alpha = 20 \text{ km}$ and for a 30 km wide glacier $L/\alpha = 1.5$. At this value rebound is 60 % of the maximum isostatic compensation (Airy compensation). For $T_e = 20 \text{ km}$, $\alpha = 50 \text{ km}$, $L/\alpha = 0.5$ and only 25 % of Airy compensation is achieved. Fig. 1(supp) shows the calculation for normalized rebound as a function of L/α based on the formula for rebound at the centre of a 2D rectangular mass anomaly(Hetenyi, 1946).

Effect of ice filled valleys

The calculations in this study are for rebound due to incision for ice-free conditions. For the free-edge models we calculate the effect of the present ice-fill within the valleys is to reduce the rebound at the

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mountain front by a maximum of 200 m. This calculation was done by subtracting a digital representation of the ice sheet elevation (www.ngdc.noaa.gov/) from our DEM for subglacial topography (Fig. 3A), and inserting this into the 3D flexure code described above.

Figure Caption: Fig. DR1 a. Cartoon showing upward bending response of lithosphere to the removal of 2D – rectangular mass of width L . P_{11} represents horizontal stress and is extensional and compressional above and below a neutral plane respectively. T_e is the effective elastic thickness.

DR1 b. A plot showing the relationship between the dimensionless ratio (L/α) to the normalized rebound (after Walcott, 1970).

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Horizontal Stresses and rebound from Glacial Unloading



