Appendix

Theoretical Basis of Electric Analog Modeling of Fluid Flow

Our electric analog model for fluid flow in faults (Fig. A) is an extension of the physical electric analog model with discrete capacitance of Santing (1963). It contains additional physical analogs for dynamic hydraulic conductivity and fracture porosity of the fault (Table A).

The physical electric analog model (Fig. A) simulates fluid flow in the plane of a vertical cross section through the geologic structure of interest. It is a two-dimensional continuum model consisting of an epoxy board with conductive coating (Fig. A). The resistivity per area R of the coating is varied across the board to represent differences in the hydraulic conductivity of geologic units. The upper and lower model boundaries contain contact bars at which the electric potential is applied (ΔV, Table A). Potential differences between the contact bars induce an electric current I across the model area by analogy with fluid flow. The lateral model boundaries are not connected to a power supply or ground so that no flow can occur across them.

Fluid Flux versus Electric Current

The scaled electric current $I\kappa\gamma$ is the direct analog of fluid volume flux through the geohydrologic section. γ reflects the scaling factor for geologic time represented by runtime seconds.

Because the simulation of faulting leads to an increase in the conductivity of the board which would imply currents translating into unreasonably high fluid supply rates, the maximum current supply to the model is limited by a fixed current supply. The current limit is set to values analogous to metamorphic fluid fluxes at the top or the interior of devolatilizing metasedimentary piles in contact or regional metamorphic environments (e.g., Walther and Orville, 1982, Peacock, 1989).

Hydraulic Conductivity versus Electric Conductivity

Darcy's law states that the specific discharge is proportional to the driving gradient of fluid pressure. For steady flow, this is analogous to Ohm's law ($I = V/R$) where R, the resistivity, is the electric parameter equivalent to the inverse of hydraulic conductivity. Thus, for steady flow in two dimensions, a cross-section through a permeable rock sequence can be represented by a conductive coating on a surface. Such coatings, spanning several orders of magnitude in their area conductivities, were created by spray painting an epoxy board with mixtures of conductor and resistor pastes (Fig. B).

By analogy to the storativity and permeability-controlled response of a porous rock to sudden pressure variations, in an electric system, the impedance Z measures time-dependent resistivity, taking into account the capacity C to store charge. The resistivity component R of the impedance of our model has been calculated from the impedance Z via

$$F_r \simeq 1/Z = \frac{1}{\sqrt{R^2 + \frac{1}{(2\pi/5RC)^2 C^2}}} \tag{1}$$

In equation 1, 1/5RC serves as a reasonable approximation of the characteristic frequency f of the circuit.

Storativity and Capacity

Since V_t, the increase in potential V which can be measured on the poles of a capacitor is proportional to the charge Q which is supplied to it, a capacitor can be used to simulate a fixed value of storativity β_c. Fixing the storativity contributes an inaccuracy in the representation of natural β_c that varies appreciably with pressure. In our model, the analog value of storativity was chosen to be accurate for the highest fluid pressure simulated in the particular model. Because the compressibility of most geological fluids decreases with increasing pressure, our β_c-values are slightly too low with respect to the negative excursions of fluid pressure simulated in the modeling. Thus, we slightly underestimate the effect of storativity on fluid pressure fluctuations.

In our electric analog model the storativity is implemented spatially by capacitors that are connected with the conductive board on a regular grid of contact points. The limitation of this discrete type of connection is that the coupling is not as direct as for a continuous electric analog of storativity.

Fluid Pressure versus Potential

In the electric analog models, the electric potential $V\kappa$ is analogous to hydraulic head (Table A). κ serves as a dimensionless scaling factor to achieve a convenient board voltage. Gravity effects, i.e., the elevation head in the geologic cross section is compensated for by subtracting the pressure exerted by the weight of the hydrostatic column from the pressure difference ΔP, simulated by applying boundary potentials to the board. The analogy between fluid pressure and electric potential is evident from a comparison between Darcy's and Ohm's law:

$$q = -F_v \nabla P \tag{2}$$

$$I = -\sigma \nabla V \tag{3}$$

where F_v is the hydraulic conductivity and I and σ are the electric current and the electrical conductivity, respectively (cf., Bear, 1972).

Fault Properties

The discontinuous change of fault conductivity and the co-seismic increase in fracture porosity in the fault have required a special model implementation.

To simulate a sudden increase in fault conductivity, 30–50 evenly spaced points along the fault were connected by flat-wire to a multi-channel switch. With respect to the average square resistivity of the model (1,266 $\Omega\,\mathrm{cm}^2$), the maximum increase in the fault conductivity that can be achieved by switching is eight orders of magnitude. This ratio has been calculated from the flatwire length, cross-sectional area, and the specific resistivity of copper (0.0175 $\Omega\,\mathrm{mm}^{-2}\,\mathrm{m}^{-1}$).

The co-seismic generation of fracture porosity along the fault is simulated with a capacitor scaled to reflect the volume storage of the newly created fracture space. However, instead of connecting this capacitor permanently to the board it is discharged and then connected to the fault with the same multichannel switch used when faulting is simulated.

Time and Scaling

A timescale is built in the electric analog model via the relationship between capacity and impedance. As shown, the scaling of these parameters depends on the physical properties and dimensions of the represented geologic structure. The model was built for time-equivalents of 10 to 100 yrs = 1 runtime second.

Model Parametrization

The described model has been built to closely represent the properties of a fine grained metamorphic rock with a small porosity at greenschist facies PT-conditions. For this purpose, the storativity (calculated for water as a pore-fluid) of Solenhofen limestone at $\approx$ 200 MPa, 400°C as determined by Fischer and Paterson (1992) was implemented. The storativity was simulated with the aid of 440 capacitors connected to the board on a rectangular array of grid points with one cm spacing. Details of the implemented geometry and parametrization are shown in Table B, and Figure 1.

Implementation Example of an Electric Analog Model

Assume a model would be designed to represent a geo-hydrologic section with a height of 740 m, a width of 600 m, unit thickness, and a range of storage capacities to be represented by single capacitors. Let us further assume that the upper boundary of this model would be at a depth of 5 km beneath the earth surface and its bulk permeability would need to be 10^{-19}m^2, because the model was aimed at representing a section with a permeability sufficiently low to allow for a fluid pressure buildup to near lithostatic pressure at a typical fluid supply rate during contact metamorphism.

For these boundary conditions, after correction for the hydrostatic gradient which has to be overcome to permit upward fluid flow, the driving pressure for flow amounts to

$$\Delta \mathrm{P} = \mathrm{P_{lith}}(5.7\,\mathrm{km}) - (\mathrm{P_{hydr}}(5\,\mathrm{km}) + \mathrm{P_h}) \approx 103\mathrm{MPa} \tag{4}$$

This gives a vertical pressure gradient ∇P of

$$\nabla \mathrm{P} = \Delta \mathrm{P}/\mathrm{z} = 1.39 \times 10^5 \mathrm{Pa\,m^{-1}} \tag{5}$$

For the desired bulk permeability k this gradient is associated with the vertical volume flux q_v

$$q_v = \frac{\mathrm{k}\nabla \mathrm{PA}}{\nu} = 1.39 \times 10^{-11} \mathrm{m^3\,m^{-2}\,s^{-1}} \tag{6}$$

This is based on a dynamic viscosity of water ν of 0.1 centipoise ($10^{-4}\mathrm{kg\,m^{-1}\,s^{-1}}$), an approximate value at 400°C, 100 MPa. The volume flux q_v allows the direct calculation of the desired bulk impedance Z of the model

$$Z = \frac{\Delta V}{q_v w d} = 1.2 \times 10^9 \Omega \tag{7}$$

The bulk capacity of the model is derived from the storage capacity β_c via the scaling relation between Δh and ΔV and the total rock volume

$$\kappa = \frac{\Delta h}{\Delta V} = 1.03 \times 10^7 \tag{8}$$

$$C = \frac{\beta_c z w d}{\kappa} \tag{9}$$

This capacity needs no further modification for the scaling of the geologic time t with the parameter γ. For instance, to achieve 1 model second equals 1 year

$$\gamma = 30,758,400 \tag{10}$$

$$Z = Z/\gamma = 39\,\Omega \tag{11}$$

For the desired time-scaling and capacity, the desired bulk resistivity of the electric analog model, which is needed to produce the conductive coating, is calculated via

$$F_r \simeq 1/Z = \frac{1}{\sqrt{R^2 + \frac{1}{(2\pi/5RC)^2 C^2}}} \tag{12}$$

Remembering that $1/5RC$ serves as a reasonable approximation of the characteristic frequency f of the circuit this equation can be solved for R by Newton Raphson iteration.

Assuming now that the limit on the fluid supply to the model should be 6.52 $\mathrm{kg\,m^{-2}\,yr^{-1}}$, and the fluid density is 730 $\mathrm{kg\,m^3}$, this amounts to a volume flux of 8.93 $\times 10^{-3}\,\mathrm{m^3}$ and the corresponding current at 1 s (model) = 1 yr (nature) should be

$$I = q_v w d = 0.54A \tag{13}$$

This is about twice the current which flows through the board when the fault is inactive. These currents are already quite high. In order to keep the currents and capacities in a range convenient for the implementation, an additional downscaling of current and capacity can be applied.

Data Acquisition

The acquired variable in the electric analog experiments is the electric potential. Potential values are collected via a detector board with 192-contact pins (Fig. A). These pins are arranged into a square array (14 $\times$ 14) with one cm spacing. The pins are gold-plated and have flexible heads which assure contact to the model when sufficient pressure is applied to the detector board.

The samples of potentials are delivered to a 16-channel 8-bit analog-digital converter in a row- by row sampling sequence. Sequential reading is achieved by multiplexer cards containing digital switches which are synchronized by a timer. The digitized samples are fed into an IBM-PC-computer where they are stored as binary files. The data acquisition hardware and software was developed by J. Lanc at the Research School of Earth Sciences, Australian National University. Complementary software was written by S.K. Matthäi.

The 8-bit digital converter translates into a sampling accuracy in the millivolt range. In practice this accuracy was reduced to tens of millivolts. In spite of this limitation, the sampling accuracy did not prove to be limiting in any of the experiments.

The time-offset between the sampling of individual pins is a potential source of error regarding the correct representation of the rapidly changing dynamic states of the model. Respective effects are circumvented by slowing down changes in potential via an adequate scaling of runtime in the experiments. This was achieved without difficulty, as it takes only 7.68 ms to sample the 192 pins. This permits the accurate measurement of potential fields even for very rapid potential changes. The maximum sampling frequency was 10 milliseconds per board sample.

The detector board has been used in three different ways to acquire data from the model. Firstly, the board has been placed on a region of interest of the (physical) electric analog model to map out the spatial variation in potential. Secondly, averages over all pins have been measured to study regional potential fluctuations in the charge region of the fault. Thirdly, individual pins have been connected to the fault to monitor potential fluctuations and gradients along the latter.

Additional References

Haar, L., Gallagher, J., Kell, G., *NBS/NRC Steam Tables*, McGraw Hill, New York (1984).

Santing, G., *Committee for Hydrological Research, Proceedings, T.N.O.*, 23–47 (1963).

Hydro-Geologic Parametrizition		Electric Analog Representation	
		Physical Dimensions	
height, width, depth [m]	z, w, d	[m]	z, w, d
cross-sectional area [m^2]	A	$w \times d$ [m^2]	A
		Time	
real time [s] (seconds)	t	scaled time [s]	$t\gamma$
		Driving Pressure versus Potential	
hydraulic head [Pa] (Pascal)	h	potential [V] (Volt)	V
head-difference across model [Pa]	Δh	potential-difference between contact bars [V]	ΔV
gradient in head [Pa m^{-1}]	$\nabla h = \frac{\partial h}{\partial z}$	potential gradient [V m^{-1}]	$\nabla V = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial z}$
		Fluid Volume versus Electric Charge	
volume [m^3]	V_f	charge [C] (Coulomb=As)	Q
volume discharge [m^{3}s^{-1}]	q_v	current [Aγ]	$I\gamma$
permeability [m^2]	k		
hydraulic conductivity [m s^{-1}]	$F_v = \frac{kg}{\mu}$	inverse impedance [Ω] [(Ohm) s^{-1}]	$1/Z = \frac{\nabla V}{I\gamma}$
storativity [m^3 Pa^{-1} m^{-3}]	β_c	capacity [F (Farad) m^{-3}]	$C = \frac{Q}{V}$
fracture voids [m^3]		charge-reservoirs of size [C]	$Q = C\Delta V$

Scaling Relations

$$\Delta h \propto \Delta V \times \kappa; \qquad \kappa = \frac{\Delta h}{\Delta V}$$

$$q_v \propto I \times \gamma$$

$$\beta_c \propto C \times \kappa$$

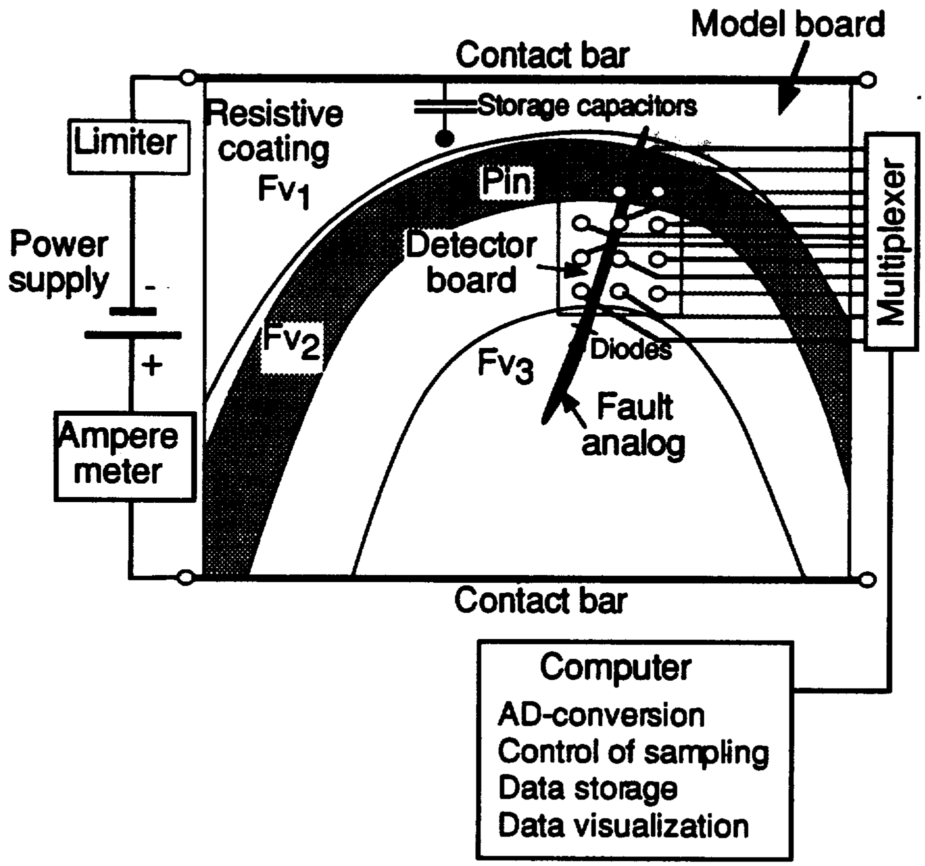
Table A: Analogies between fluid flow and electric charge transport. SI units in square brackets. μ is the dynamic viscosity of the fluid and g is the acceleration of gravity (9.81 m^{2}s^{-1}). γ and κ are scaling factors for time and flux, respectively.

Geo-hydrologic Section			EA3 Model		
Parameter	Value		Parameter	Value	
Subsurface Depth	9,000	m			
Height	1,000	m	z	220	mm
Width	1,000	m	w	220	mm
Thickness	1	m	d	0.5	mm
Rock density	2,750	$\mathrm{kg\,m^{-3}}$			
Fluid density	832	$\mathrm{kg\,m^{-3}}$			
Kinematic viscosity	0.0001	$\mathrm{kg\,m^{-1}\,s^{-1}}$			
Geothermal Gradient	40	$\mathrm{^\circ C\,km^{-1}}$			
Basal temperature	400	°C			
Time	1	yrs	t	1	s
P_f at base	275	MPa	V_b	10	s
P_f at top	198	MPa	V_t	0	V
P_h-water column	7.6	MPa			
avg. permeability	10^{-20}	$\mathrm{m^2}$	ξ		
Hydraulic conductivity	10^{-15}	$\mathrm{m\,s^{-1}}$	$1/Z$	2.7×10^{-5}	$1/\Omega$
Basal volume influx	0.00021	$\mathrm{m^3\,m^{-2}\,yr^{-1}}$	$I\kappa$	0.02	mA
Basal mass influx	0.1776	$\mathrm{kg\,m^{-2}\,yr^{-1}}$			
Max. mass influx	0.3	$\mathrm{kg\,m^{-2}\,yr^{-1}}$	$\simeq I_{max}$	0.034	mA
Storage capacity (log)	-9.85	$\mathrm{m^3\,Pa^{-1}}$	C	97,020	μF
			Fault Properties		
Fault Length	272	m	F_l	60	mm
Fault Width	9	m	F_w	2	mm
Fracture porosity Φ	3.0	%	$\simeq C_F$		
Seismic Φ-increase	20	%	$\simeq C_n$	4,900	μF
Hydr. conductivity	10^{-15}	$\mathrm{m\,s^{-1}}$	$1/Z_F$		
Failure h-conductivity	10^{-8}	$\mathrm{m\,s^{-1}}$	$1/Z_t$		

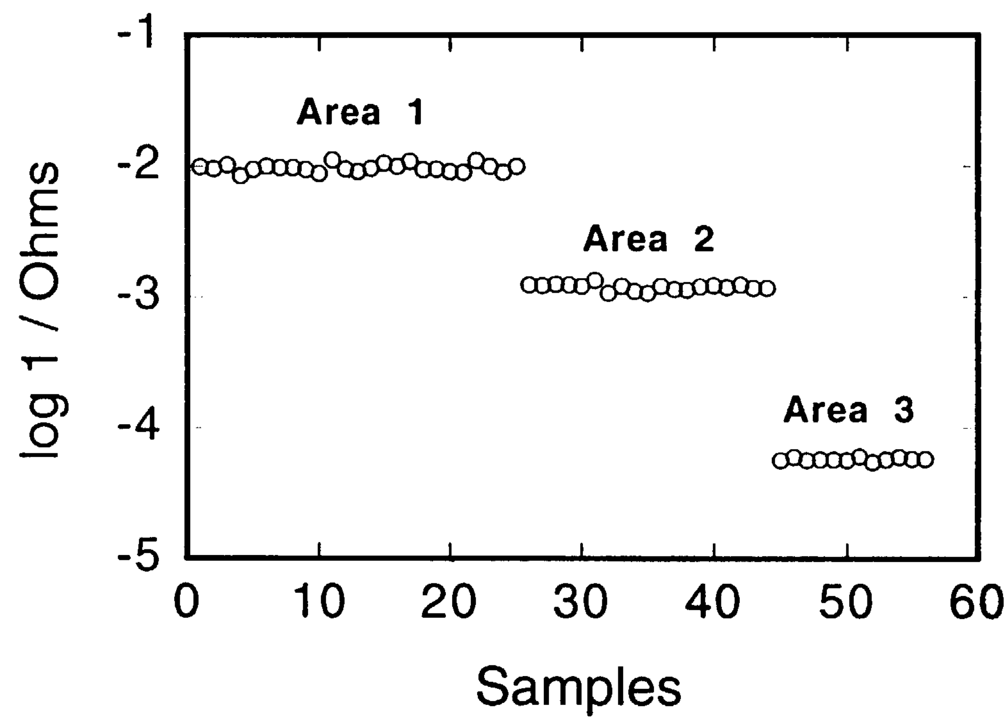
Table B: Parametrization of electric analog model EA3. The storage capacity represents an intermediate value determined in experiments on Solenhofen limestone at 400°C, 200 MPa (Fischer and Paterson, 1992). This storativity was re-calculated for an aqueous fluid using water compressibilities calculated with the equation of state by Haar et al. (1984).

Figure A: Electric analog model for fluid flow in faults. Model components and their interfacing. Experimental setup for the determination of flow patterns (potential gradients) during faulting.

Figure B: Homogeneity and accuracy of the scaling of the electric conductivity of the resistive coatings which were applied to the electric analog models. Measurements were taken with an Ohm-meter over 2-cm distances. The average square resistivity is 1,266 $\Omega\,\mathrm{cm}^2$.



Matthai & Fischer, Appendix Figure A



Area 1
Area 2
Area 3
log 1 / Ohms
-1
-2
-3
-4
-5
0
10
20
30
40
50
60
Samples