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Title of article Ejection of rock fragments from planetary bodies

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Mathematical Description of Stress Wave Interference

The derivation of the equations in this addendum and the assumptions behind several approximations, are described in more detail in Melosh (1984a). The following simplified sketch will allow the reader to follow the main lines of the argument without getting bogged down in too much detail.

Experience from underground nuclear tests (Perret and Bass, 1975) and several shock wave attenuation computations (Ahrens and O'Keefe, 1977; Orphal et al., 1980) show that the maximum pressure in the vicinity of an impact falls roughly as the inverse square of the distance R from the equivalent center for distances R larger than a few projectile radii ($D/2$). This pressure is denoted $P_{\text{free}}(R)$:

$$P_{\text{free}} = P_i \left(\frac{D}{2R} \right)^2 \quad (1A)$$

where P_i is the maximum pressure experienced in the impact. It is given roughly by the planar impact approximation (Shoemaker, 1960). If both projectile and target have the same uncompressed density, ρ_0 , P_i is

$$P_i = \rho_0 c_L U/2 \quad (2A)$$

where the (variable) shock velocity is replaced by the constant longitudinal sound speed c_L in the target (see Melosh, 1984, for the rationale).

Figure 3 demonstrates that the maximum and minimum pressures P_{max} and P_{min} , respectively, experienced at a point at epicentral range S and depth z from the impact site depend upon the time delay Δ between the arrival of the direct and reflected waves. This delay (for $z \ll D$) is

$$\Delta \approx \frac{2Dz}{c_L R} \quad (3A)$$

where $R = \sqrt{S^2 + D^2}$. The pressures are then given by

$$P_{\max} = \frac{\Delta}{\tau} P_{\text{free}}(R) \quad (4A)$$

$$P_{\min} = -\frac{\Delta}{\beta\tau} P_{\text{free}}(R) \quad (5A)$$

The negative sign in (5A) indicates that the pressure is tensile.

The ejection velocity is computed from the vector sum of particle velocities in the direct and tensile waves. The radial particle velocity in the compressive wave is $v_P = P_{\text{free}} / \rho_0 c_L$, by the second Hugoniot equation. The vertical component is $v_{\perp} = Dv_P / R$. Adding the component from the tensile wave yields

$$v_e = \frac{2D}{\rho_0 c_L R} P_{\text{free}} \quad (6A)$$

In this simple model the ejection angle is vertical: the observed ejection angle of near 45° is a result of P- and S-wave interactions described in Melosh (1984). Nevertheless, this model gives an excellent estimate of the ejection speed, if not its direction.

The spall thickness z_s is computed from the depth at which the tensile wave first reaches the dynamic tensile strength T . Numerical computations (Melosh, in preparation) show that, due to the finite time before fracture occurs in the Grady-Kipp model, the actual spall thickness is about twice that computed from the above simple prescription. This factor of two is incorporated in equation (4). Thus, setting equation (5A) equal to $-2T$ and

using equation (3A), we find

$$z_s \approx \left\{ \frac{T}{P_{\text{free}}(R)} \right\} R \quad (7A)$$

where $c_L \beta \tau \approx D$ has been used to simplify the equation. Elimination of P_{free} from (7A) using (6A) yields equation (4) of the text.