STRESS FUNCTIONS

BGSA DATA REPOSITORY of Weijermars, R. , Estimation of paleostress orientation within deformation zones between two mobile plates

The regional state of stress in a rock volume can best be visualized as stress trajectories, and the most elegant tool to formulate the spatial state of stress is in terms of Airy's stress function ϕ. The use of Airy's stress function has been demonstrated in several analytical studies of particular geological fault patterns (Anderson, 1951; Hafner, 1951; Odé, 1957; Sanford, 1959; Couples, 1977; Muller and Pollard, 1977; Spencer and Chase, 1989). However, a method to characterize the stress trajectory pattern in three orthogonal planes with a set of three stress functions is first introduced here. This allows the description of stress for the following three-dimensional cases (figures in brackets refer to those in Weijermars, in review):

A. Reverse faulting and homogeneous shortening (figs. 1a, 2a, 2c)

B. Normal faulting and homogeneous extension (figs. 2b, 2d)

C. Shortening deformation zone by horizontal pure shear (figs. 5a,5c).

D. Horizontal pure shear with extension of deformation zone (figs. 10a, 10c)

E. Horizontal simple shear (figs. 5b, 5d, 10b, 10d)

5c)

Cases A-E are discussed in turn below after a short description of Airy's stress function, ϕ , a potential field with units: Pa m^2. This function describes the deviatoric stresses arising from surface forces in addition to body stresses in the following fashion:

$$\tau_{zz} = \partial^2\phi/\partial x^2 \qquad (1a)$$

$$\tau_{xx} = \partial^2\phi/\partial z^2 \qquad (1b)$$

$$\tau_{xz} = -\partial^2\phi/\partial x\partial z \qquad (1c)$$

The definition of ϕ is so elegantly constructed that the equilibrium equations are automatically satisfied in plane stress problems, i.e.:

$$\partial\tau_{xx}/\partial x + \partial\tau_{xz}/\partial z = 0 \qquad (2a)$$

$$\partial\tau_{zz}/\partial z + \partial\tau_{xz}/\partial x = 0 \qquad (2b)$$

An appropriate stress function ϕ also complies with the condition of strain compatibility to warrant continuity of deformation:

$$\text{lap lap } \phi = 0 \qquad (3)$$

The stress trajectory inclination β with respect to the X-axis is determined by the stress function through the expression (neglecting body forces, because their stress trajectories comprise a pattern of orthogonal straight lines):

$$\tan 2\beta = -(2\partial^2\phi/\partial x\partial z) \;/\; [(\partial^2\phi/\partial z^2)-(\partial^2\phi/\partial x^2)] \qquad (4)$$

The stress fields described by the sets of stress functions derived below are termed here uniform, although the stress field in the vicinity of the fault surfaces may be modified after the initial failure (Chinnery, 1964, 1966a,b). Nonetheless, it may be assumed that components of the bulk stress tensor remain constant in space. The condition P=0 probably is only strictly valid in nature for simple shear in the horizontal plane (case E). Converging and diverging plates with oblique motion (cases C and D), constrained here to deformation in the horizontal plane by specially defined boundary conditions, are likely to involve transpression and transtension, respectively. If this were the case, the confinig pressure, P, will not be zero as followed here, but involves overpresssure in transpression and underpressure in transtension. These advanced cases of three-dimensional deformation will be modelled elsewhere (work in preparation).

A. Reverse faulting and homogeneous shortening (Figs. 1a, 2a, 2c)

The stress field of these cases involve 1) biaxial deviatoric stress causing bulk pure shear in the XZ-plane, 2) uniaxial compression in the XY-plane and 3) uniaxial tension in the YZ-plane (Fig. 1a). Compression is in the X-direction and tension in the Z-direction. The stress in each of the three planes may be formulated by three complementary stress functions $\phi(x,z)$, $\phi(x,y)$ and $\phi(y,z)$. If the total tectonic stress is $\sigma_{xx} = \sigma_1$ then the three stress functions are:

$$\phi(x,z) = (\sigma_1/4)\ (z^2 - x^2) \qquad (5a)$$

$$\phi(x,y) = (\sigma_1/4)\ y^2 \qquad (5b)$$

$$\phi(y,z) = -(\sigma_1/4)\ y^2 \qquad (5c)$$

Differentiation of equations (5a-c) according to (1a-c) yields all relevant components of the deviatoric stress tensor, i.e.:

$\tau_{xx} = \sigma_1/2$	$\tau_{zz} = -\sigma_1/2$	$\tau_{xz} = 0$	from (5a)
$\tau_{xx} = \sigma_1/2$	$\tau_{yy} = 0$	$\tau_{xy} = 0$	from (5b)
$\tau_{zz} = -\sigma_1/2$	$\tau_{yy} = 0$	$\tau_{yz} = 0$	from (5c)

Differentiation of equations (5a-c) according to expressions (2a,b) and (3) reveals that both equilibrium and strain compatibility conditions are fullfilled. Differentiation of equation (4) shows that all stress trajectories are parallel to the coordinate axes. It has been shown in section 3 that $P = \sigma_1/2$.

B. Normal faulting and homogeneous extension (Figs. 2b, 2d)

These cases involve 1) biaxial deviatoric stress causing bulk pure shear in the XZ-plane, 2) uniaxial tension in the XY-plane, and 3) uniaxial compression in the YZ-plane (Fig. 1b). Tension is in the X-direction and compression in the Z-direction. The stress in each of the three planes may, again, be formulated by three complementary stress functions $\phi(x,z)$, $\phi(x,y)$ and $\phi(y,z)$. If the total tectonically applied stress is $\sigma_{xx} = -\sigma_1$ then the three stress functions are:

$$\phi(x,z) = (\sigma_1/4)\ (x^2 - z^2) \qquad (6a)$$

$$\phi(x,y) = -(\sigma_1/4)\ y^2 \qquad (6b)$$

$$\phi(y,z) = (\sigma_1/4)\ y^2 \qquad (6c)$$

Differentiation of equations (6a-c) according to (1a-c) yields all relevant components of the deviatoric stress tensor, i.e.:

$\tau_{xx} = -\sigma_1/2$ $\quad \tau_{zz} = \sigma_1/2$ $\quad \tau_{xz} = 0$ $\quad$ from (6a)

$\tau_{xx} = -\sigma_1/2$ $\quad \tau_{yy} = 0$ $\quad \tau_{xy} = 0$ $\quad$ from (6b)

$\tau_{zz} = \sigma_1/2$ $\quad \tau_{yy} = 0$ $\quad \tau_{yz} = 0$ $\quad$ from (6c)

Differentiation of equations (6a-c) according to expressions (2a,b) and (3) reveals that both equilibrium and strain compatibility conditions are fullfilled. Differentiation of equation (4) shows that all stress trajectories are parallel to the coordinate axes. It has been shown in section 3 that $P = -\sigma_1/2$.

C. Shortening deformation zone by horizontal pure shear (Figs. 5a, 5c)

This case involves 1) biaxial deviatoric stress causing pure shear in the XZ-plane, 2) uniaxial tension in the XY-plane, and 3) uniaxial compression in the YZ-plane (Fig. 1c). Compression is in the Z-direction and tension in the X-direction. If the total tectonically appplied stress is $\sigma_{zz} = \sigma_1$ then the stress functions $\phi(x,z)$, $\phi(x,y)$, and $\phi(y,z)$ in each of the three planes are:

$$\phi(x,z) = -(\sigma_1/2)\ (z^2 - x^2) \qquad (7a)$$

$$\phi(x,y) = -(\sigma_1/2)\ y^2 \qquad (7b)$$

$$\phi(y,z) = +(\sigma_1/2)\ y^2 \qquad (7c)$$

Differentiation of equations (7a-c) according to (1a-c) yields all relevant components of the deviatoric stress tensor, i.e.:

$\tau_{xx} = -\sigma_1$	$\tau_{zz} = \sigma_1$	$\tau_{xz} = 0$	from (7a)
$\tau_{xx} = -\sigma_1$	$\tau_{yy} = 0$	$\tau_{xy} = 0$	from (7b)
$\tau_{zz} = \sigma_1$	$\tau_{yy} = 0$	$\tau_{yz} = 0$	from (7c)

Differentiation of equations (7a-c) according to expressions (2a,b) and (3) reveals that both equilibrium and strain compatibility conditions are fullfilled. If the major and minor principal deviatoric stress axes are both in the horizontal XZ-plane it follows from the condition of plane strain that all vertical stresses are zero, i.e. both $\sigma_{yy} = 0$ and $\tau_{yy} = 0$. Substitution in $\tau_{yy} = \sigma_{yy} - P$ yields $P = 0$.

D. Horizontal pure shear with extension of deformation zone (Figs. 10a, 10c)

This case involves 1) biaxial deviatoric stress causing pure shear in the XZ-plane, 2) uniaxial tension in the XY-plane, and 3) uniaxial compression in the YZ-plane (Fig. 1d). Compression is in the X-direction and tension in the Z-direction. If the total tectonic stress is $\sigma_{zz} = \sigma_1$ then the stress functions $\phi(x,z)$, $\phi(x,y)$, and $\phi(y,z)$ in each of the three planes are:

$$\phi(x,z) = (\sigma_1/2)\ (z^2 - x^2) \qquad (8a)$$

$$\phi(x,y) = (\sigma_1/2)\ y^2 \qquad (8b)$$

$$\phi(y,z) = -(\sigma_1/2)\ y^2 \qquad (8c)$$

Differentiation of equations (8a-c) according to (1a-c) yields all relevant components of the deviatoric stress tensor, i.e.:

$\tau_{xx} = \sigma_1$	$\tau_{zz} = -\sigma_1$	$\tau_{xz} = 0$	from (8a)
$\tau_{xx} = \sigma_1$	$\tau_{yy} = 0$	$\tau_{xy} = 0$	from (8b)
$\tau_{zz} = -\sigma_1$	$\tau_{yy} = 0$	$\tau_{yz} = 0$	from (8c)

Differentiation of equations (8a-c) according to expressions (2a,b) and (3) reveals that both equilibrium and strain compatibility conditions are fullfilled. It follows from the condition of plane strain that all vertical stresses are zero, i.e. both $\sigma_{yy} = 0$ and $\tau_{yy} = 0$. Substitution in $\tau_{yy} = \sigma_{yy} - P$ yields $P = 0$.

E. Horizontal simple shear (Figs. 5b, 5d, 10b, 10d)

This case includes Anderson's (1951) condition for wrench faulting. The stress field involves: 1) bulk simple shear in the XZ-plane (Fig. 1e). If the total tectonic stress is σ_1 then the stress functions $\phi(x,z)$, $\phi(x,y)$, and $\phi(y,z)$ in each of the three planes are:

$$\phi(x,z) = -\sigma_1\ xy \qquad (9a)$$

$$\phi(x,y) = 0 \qquad (9b)$$

$$\phi(y,z) = 0 \qquad (9c)$$

Differentiation of equations (9a-c) according to (1a-c) yields all relevant components of the deviatoric stress tensor, i.e.:

$\tau_{xx} = 0$	$\tau_{zz} = 0$	$\tau_{xz} = \sigma_1$	from (9a)
$\tau_{xx} = 0$	$\tau_{yy} = 0$	$\tau_{xy} = 0$	from (9b)
$\tau_{zz} = 0$	$\tau_{yy} = 0$	$\tau_{yz} = 0$	from (9c)

Differentiation of equations (9a-c) according to expressions (2a,b) and (3) reveals that both equilibrium and strain compatibility conditions are fullfilled. Differentiation of equation (4) shows that all stress trajectories are parallel to the coordinate axes, except for σ_1-trajectories which are within the XZ-plane at 45° to the X-axis. It follows from the condition of plane strain that all vertical stresses are zero, i.e. both $\sigma_{yy} = 0$ and $\tau_{yy} = 0$. Substitution in $\tau_{yy} = \sigma_{yy} - P$ yields $P = 0$.

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FIGURE CAPTION

Fig. 1: Sketches showing deviatoric stress trajectories in three orthogonal planes for the cases A-E discussed in the text. These are a) reverse faulting and homogeneous shortening, b) normal faulting and homogeneous extension, c) horizontal pure shear with shortening of deformation zone, d) horizontal pure shear with extension of deformation zone, and e) horizontal simple shear.

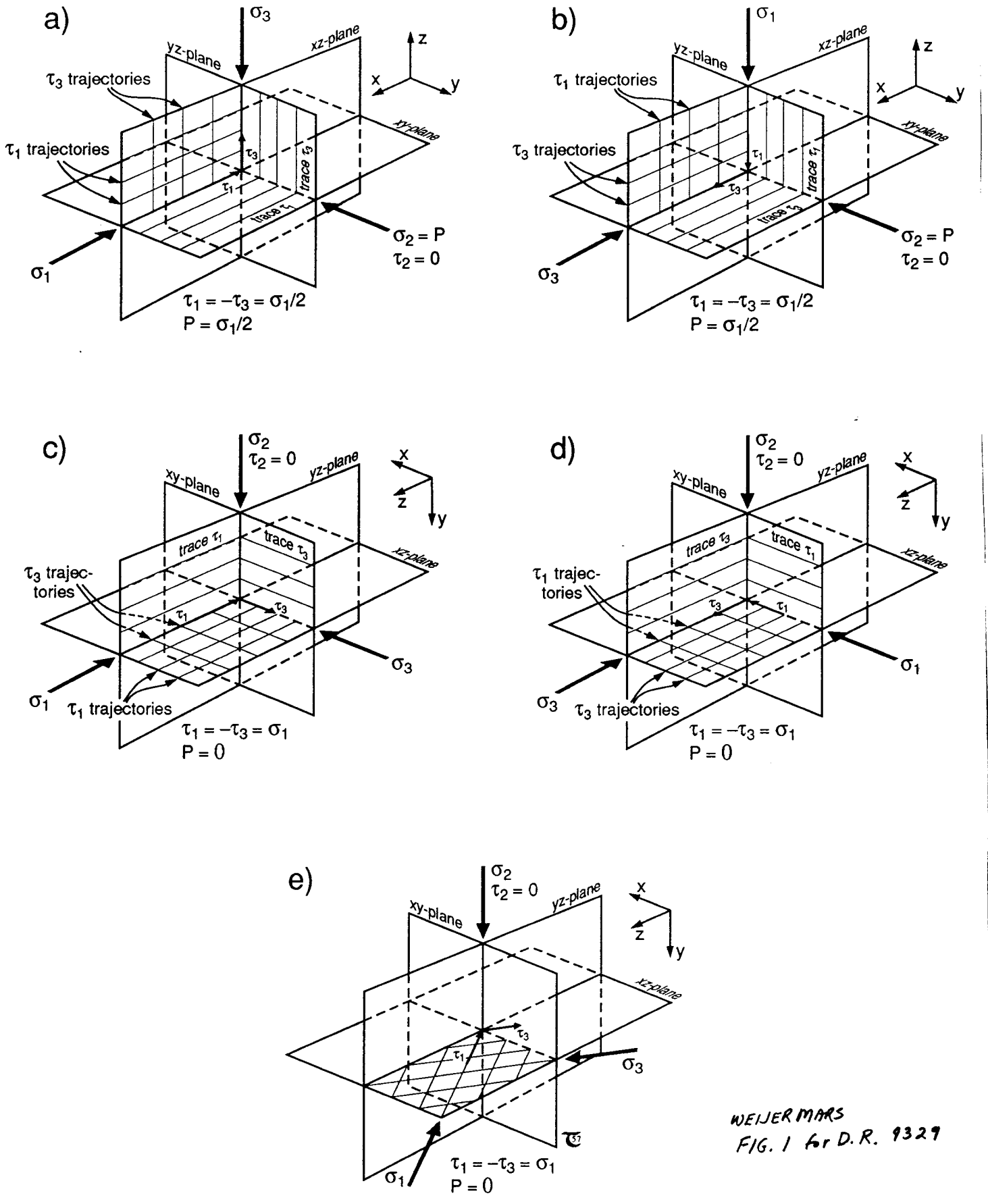

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FIG. 1 for D.R. 9329