APPENDIX: Heat Conductive Model

Kevin P. Furlong and Jacqueline E. Huntoon[1]

Department of Geosciences, Pennsylvania State University,

University Park, PA 16802

Finite Difference Approximation

The thermal evolution of the crust was evaluated using a numerical approximation (finite difference) of equation 2. To implement the model, boundary conditions of surface temperature and surface heat flow are specified. Temperatures are determined throughout the crust on a discrete grid of positions at a regular interval of time steps. The upper 30 km of the crust was modeled using a spatial discretization of 0.1 km (100 m), and time steps of 0.1 Ma (100,000 years).

The components of equation 2 can be rewritten in finite difference form:

Time Derivative (Left-Hand Side)

$$\frac{\partial T}{\partial t} = \frac{1}{k} [T_i^{j+1} - T_i^j] \qquad (2.1)$$

and Spatial Derivative (Right-Hand Side)

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{h^2}[\phi(T_{i+1}^{j+1} - 2T_i^{j+1} + T_{i-1}^{j+1}) + (1-\phi)(T_{i+1}^j - 2T_i^j + T_{i-1}^j)] \qquad (2.2)$$

In these discretizations, k represents the time step, h is the space step, and the indices i and j refer to specific space and time steps, respectively. This discretization of the right-hand side (2.2) uses a weighted average of temperatures at the current (j) time step and at the desired next time step (j+1). ϕ is the weighting function and, for the implicit method used here, was chosen to be $\frac{1}{2}$. The spatial discretization uses temperature values at a central and adjacent nodes (central difference) at both the current and future time step. The use of multiple spatial nodes at the future time step makes the solution approach implicit which produces an unconditionally stable solution. Equations 2.1 and 2.2 are combined with equation 2 to generate the finite difference approximation solved :

$$(1+r\alpha_i)T_i^{j+1}-\frac{r\alpha_i}{2}[T_{i+1}^{j+1}+T_{i-1}^{j+1}] = (1-r\alpha_i)T_i^j+\frac{r\alpha_i}{2}[T_{i+1}^j+T_{i-1}^j]+(\alpha_i k)\frac{A_i}{K_i} \quad (2.3)$$

where $r = \frac{k}{h^2}$, α_i, A_i, and K_i are thermal diffusivity, heat production, and thermal conductivity (respectively) at the i^{th} spatial node. All of the terms on the right-hand side of (2.3) are known quantities (values at present time step), while the terms on the left-hand side are unknowns to be determined. Equation 2.3 can be described as a tridiagonal matrix equation which can be easily solved at each time step using standard matrix solvers.

In the model, we start with a specified temperature structure at the beginning of the model period and step through time, using the previous values via equation 2.3 to determine temperature values at the new time step. This process is repeated throughout the time period of interest. Throughout the model time period the boundary conditions of specified surface temperature and heat flow are maintained.

Incorporation of Tectonic History

In order to simulate the temperature history throughout a geologic period involving the possibility of sedimentary burial, erosion, and thrusting, the model used here explicitly incorporated the thermal effects of these processes in simulating the temperature history. Sedimentary burial is simulated by adding at appropriate time steps packets of material of specified thermal conductivity/diffusivity, emplaced at surface temperature. Since the spatial discretization is 100 m, each packet represents the equivalent of a 100 m thick sedimentary strata. Multiple smaller sedimentary units can be combined into 100 m units. Erosion, as the inverse of sedimentation, is simulated by the removal of 100 m packets. In both cases the thermal effects of the burial and/or erosion propagate through the crust according to the physics of equation 2.

Although not used in this study, the modeling algorithm can also incorporate the effects of thrusting. This is done through the juxtaposition of nodal information in line with the geometry of the thrust fault. Thrusting events can be combined with erosion and sedimentation to simulate arbitrarily complex tectonic events.

Although the initial specified boundary conditions are surface temperature and heat flow, the use of surface heat flow throughout the modeling of tectonic activity would not replicate geologic thermal processes. To overcome this potential problem, the specified surface heat flow is converted to an equivalent basal heat flow after accounting for all heat sources within the crust. This basal flux is used as the boundary condition throughout the modeling period.

---

[1]Present address: Department of Geological Engineering, Geology and Geophysics, Michigan Technological University, Houghton, MI 49931.