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Title of article Use of longitudinal strain in identifying driving and resisting elements of landslides

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APPENDIX

DERIVATION OF FORMULAS FOR ESTIMATING THE SLOPE OF THE FAILURE SURFACE AT THE BOUNDARY BETWEEN DRIVING AND RESISTING ELEMENTS OF A LANDSLIDE

To derive the formula for the slope of the failure surface at the boundary between self-supporting and inherently unstable ground shown as equation 1 in the text, we begin with the equations for horizontal and vertical equilibrium. The equations of equilibrium are equivalent to those used in generalized methods of stability analysis (Morgenstern and Price, 1965; Janbu, 1973). The equations of force equilibrium can be derived by setting the sums of forces parallel with x and z, (Fig. 5), equal to zero and simplifying,

$$\frac{dE}{dx} = \sigma_{ns} - \sigma_{nn}\frac{d\xi}{dx} \quad ; \tag{A1}$$

$$\frac{dT}{dx} + \gamma_t D = \sigma_{nn} + \sigma_{ns}\frac{d\xi}{dx} \quad . \tag{A2}$$

In equations A1 and A2, E and T are the resultant horizontal and (vertical) shear forces per unit width acting on a vertical plane through the landslide, γ_t is the total unit weight of the material, D is the vertical distance from the slip surface, $z=\xi(x)$, to the ground surface, $z=\zeta(x)$, ($D = \zeta - \xi$) and σ_{nn} and σ_{ns} are the total normal and shear stresses acting at the slip surface, respectively. Normal stresses are assumed positive in compression. The angle θ (Fig. 5) is positive if measured counterclockwise from the horizontal to the slip surface. By this convention, $d\xi/dx$ (the slope of the slip surface) equals $\tan(\theta)$.

The shearing resistance at the slip surface is determined by the Mohr-Coulomb failure criterion for effective stress (Lambe and Whitman, 1969):

$$|\sigma_{ns}| = c' + \sigma'_{nn} \tan(\phi') \quad . \tag{A3}$$

In equation A3, c' is the cohesion and ϕ' is the angle of friction, both for effective stress, and σ'_{nn} is the effective normal stress. We assume that the shear stress equals the shear strength along the entire slip surface of an *active* landslide. The standard definition for effective stress in soil mechanics is (see, for example, Lambe and Whitman, 1969, p. 241):

$$\sigma'_{nn} = \sigma_{nn} - u \quad . \tag{A4}$$

Equation 1 in the text is derived by setting dE/dx in equation A1 equal to zero, solving for $d\xi/dx$, and substituting equation A3 for the shear stress. The approximation for the total normal stress used in the text, $D\gamma_t \cos^2\theta$), is derived by solving equations A1 and A2 for σ_{nn}, and letting dE/dx and dT/dx equal zero.

Derivation of equation (2) in the text begins with the definition of E. The resultant longitudinal total force per unit width, E, is determined by summing infinitesimal longitudinal forces (also per unit width), $[u + \sigma'_{xx}]dz$, acting on a vertical plane through the landslide,

$$E = \int_{\xi}^{\zeta} [u + \sigma'_{xx}] \, dz \quad . \tag{A5}$$

Next we assumed that the horizontal effective stress was proportional to the effective vertical stress due to the overburden,

$$\sigma'_{xx} = k[(\zeta - z)\gamma_t - u] \quad . \tag{A6}$$

In equation A6, k is the coefficient of longitudinal effective stress, assumed constant with depth, and the effective vertical stress is approximated by the quantity in square brackets. This k is similar to the coefficients used in soil mechanics to compute lateral earth pressures for active, at rest, and passive conditions.

We substituted equation A6 into equation A5, differentiated the resulting formula for E with respect to x (using Leibnitz' rule), set dE/dx equal to zero, and solved for $d\xi/dx$ to obtain the following:

$$\frac{d\xi}{dx} = \frac{1}{[u + k[(\zeta - z)\gamma_t - u]]_\xi} \bullet \left\{ \int_\xi^\zeta \frac{\partial(u + k[(\zeta - z)\gamma_t - u])}{\partial x}\, dz + \frac{d\zeta}{dx}\, [u + k[(\zeta - z)\gamma_t - u]]_\zeta \right\} \quad . \tag{A7}$$

The notation $[\,]_\xi$ in equation A7 denotes that the quantity inside square brackets is evaluated at ξ.

Next, we evaluated the integral and the quantities in square brackets in A7 and combined terms,

$$\frac{d\xi}{dx} = \frac{1}{[(1-k)\,u|_{\xi} + k\gamma_t D]}\left\{ (1-k)\int_{\xi}^{\zeta}\frac{\partial u}{\partial x}\,dz \right.$$
$$+ \left(k\gamma_t D + (1-k)\,u|_{\zeta}\right)\frac{d\zeta}{dx}$$
$$\left. + \frac{dk}{dx}\left(\frac{\gamma_t D^2}{2} - \int_{\xi}^{\zeta} u\,dz\right)\right\} \quad . \qquad \text{(A8)}$$

In equation A8, $u|_{\xi}$ denotes that u is evaluated at ξ.

Finally, we added and subtracted $(d\zeta/dx)(1-k)u|_{\xi}$ inside the braces in order to bring the slope of the ground surface, $d\zeta/dx$, outside the braces as a separate term,

$$\frac{d\xi}{dx} = \frac{d\zeta}{dx} + \frac{1}{[(1-k)\,u|_{\xi} + k\gamma_t D]}\left\{ (1-k)\int_{\xi}^{\zeta}\frac{\partial u}{\partial x}\,dz \right.$$
$$+ \left(u|_{\zeta} - u|_{\xi}\right)(1-k)\frac{d\zeta}{dx}$$
$$\left. + \frac{dk}{dx}\left(\frac{\gamma_t D^2}{2} - \int_{\xi}^{\zeta} u\,dz\right)\right\} \quad . \qquad \text{(A9)}$$

Equation A9 indicates that the slope of the failure surface, $d\xi/dx$, at the boundary between driving and resisting elements of the landslide (where dE/dx is zero) is equal to the slope of the ground surface, $d\zeta/dx$, plus some corrections for the distribution of pressure head and the horizontal effective stress. Equation 2 in the text is obtained by letting k be constant in equation A9, and neglecting u. The pore pressure, u, changes $\partial\xi/\partial x$ in equation A9 by less than $\pm 0.12(\partial\zeta/\partial x)$ if the vertical hydraulic gradient, $1+(1/\gamma_w)(\partial u/\partial z)$, is relatively small so that $-0.4 \leq 1+(1/\gamma_w)(\partial u/\partial z) \leq 0.4$; the slope of the ground surface is fairly smooth so that $\partial u/\partial x \cong \gamma_w(\partial\zeta/\partial x)$; and the earth-pressure coefficient takes on moderate values typical for cohesive soils, $0.6 \leq k \leq 1.8$.