Elicitation of error distributions in cross section interpretation – Manchester Rockhead

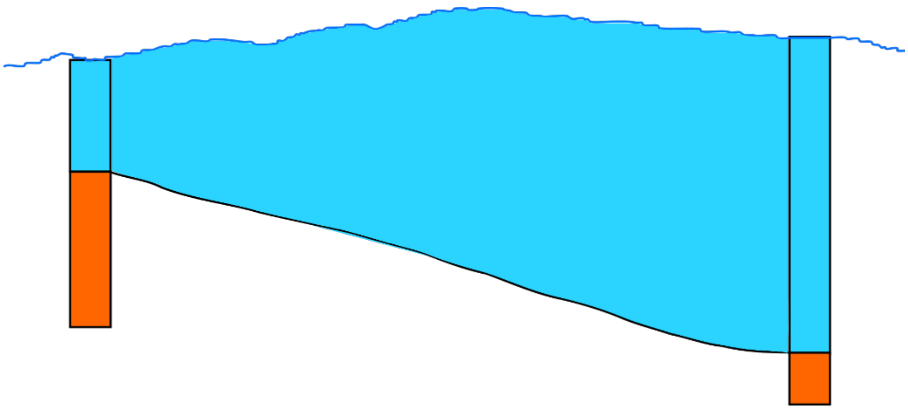
Introduction

This exercise is part of a project to quantify the uncertainties in the cross-section interpretations that underpin 3D geological models made by methods such as GSI3D. As part of the project we have undertaken experiments on cross sections with many observations. This is a powerful approach, but labour-intensive, and restricted to geological settings where we have intensively-observed cross sections. The objective of this elicitation exercise is to determine whether expert elicitation is feasible as an alternative or complementary method to quantify uncertainty. If the results are positive then the objective is to carry out more elicitations to determine the uncertainty over more geological situations.

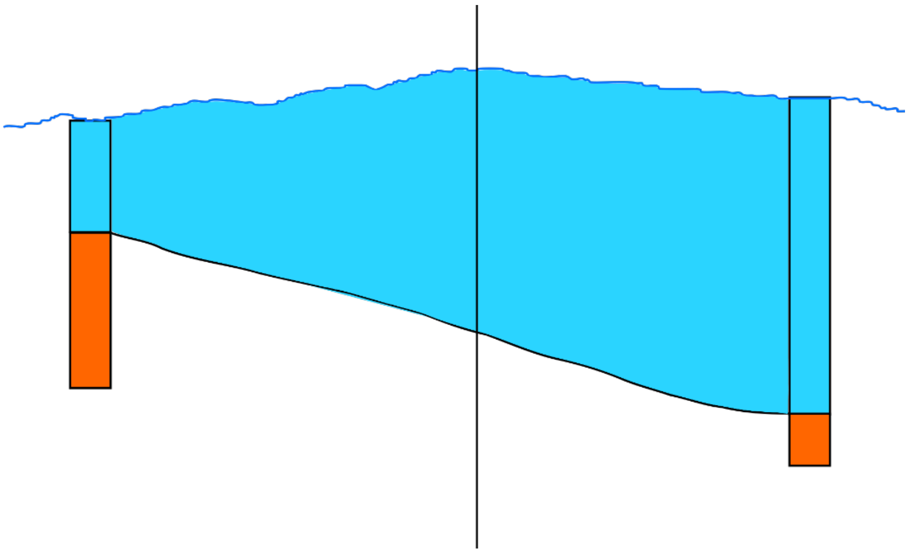
Someone with expertise in geology, and experience in interpreting cross-sections is aware of uncertainty. They have a tacit mental model of that uncertainty which they would draw on, for example, if offering advice to a user on interpretations of the section or associated models. Expert elicitation is a process that attempts to apply numbers to an expert's extract of this mental model in a more general useable form. In this exercise we will use the Sheffield Elicitation Framework (SHELF), a commonly-used framework which dictates a standardised set of procedures to follow to ensure that the results are as reliable and accurate as possible. SHELF and similar frameworks have been used to elicit models of uncertainty in medicine, engineering and many other applications. The SHELF procedure starts with individual interpretations of uncertainty in a particular scenario, but aims to arrive at a consensus view.

The situation to consider

We shall consider a number of situations in which an interpretation has been made of borehole observations of a contact between two units along a cross-section, and we then consider the position of the contact in a notional borehole somewhere inbetween. The figure below shows two boreholes on a cross section, the ground surface is shown between them.



Now assume that we sample the interpreted cross section at a location between these two boreholes. In the figure below we show the notional borehole marked with the black line.



The modeller has inferred an elevation for the contact at the position of the notional borehole. This represents their best interpretation of the available data, but it is uncertain. There are many sources of uncertainty. In this exercise we do NOT consider the following:

i. Uncertainty about the true position of the boreholes (we assume that this is known without error).
ii. Uncertainty about the true elevation of the boreholes
ii. Uncertainty about the interpretation of the units within the boreholes.

We are concerned with the following sources of uncertainty:

i. Generalization: we represent the surface by a line with a certain smoothness, but that may not be realistic, at least not in all settings.

ii. We assume that the surface behaves in certain ways because of its geological properties (probability of faulting, depositional environment, erosional environment etc) but how reliable are these assumptions?

iii. Even if the conceptual model behind the interpretation is sound, and the degree of generalization is sensible, the limited observations (given their density, proximity to the notional borehole etc) and the model may be consistent with a range of possible interpretations. The selected interpretation may be most plausible, but others are possible.

In the elicitation we consider 100 notional instances of the illustrated setting that are identical in all respects. In each instance the interpretation is the same (the geologist has identical information available to them) but there will be a different error in the interpretation, resulting from the factors listed above. The 100 instances will sample the range of possible forms of the surface consistent with its geology and the constraining observations. What we want to elicit is what the statistical distribution of those errors looks like, in so far as it is reflected in the expert geologist's mental model of uncertainty.

Probability distributions

As you are probably aware there are a wide variety of reasons why differences in interpretations occur in geology. These could be ranging in magnitude from small differences due to how the lines in a cross section were drawn, up to large differences resulting from differing conceptual ideas of what has taken place. You will be asked to summarise these by creating a probability distribution bit by bit based on your assessment of probabilities throughout this experiment.

One of the simplest ways of considering the idea of a probability distribution is to consider a dartboard, at which 100 people have been asked to throw one dart. You would expect a distribution similar to Figure 1:

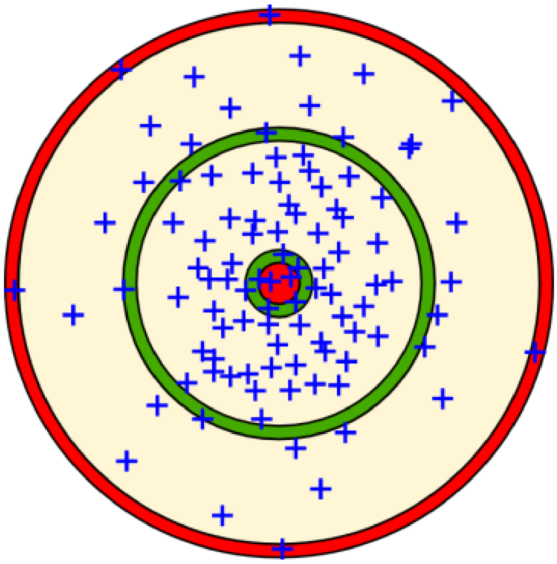


Figure 1: A hypothetical dartboard with the locations of 100 thrown darts marked with a blue cross

Now imagine that only the horizontal position of the darts was important, you can then use their spread to work out some key parameters and to plot a probability distribution. The most obvious parameters are the maximum and minimum. These are the locations of the highest and lowest possible values respectively, and represent the two ends of the distribution – in this case the two edges of the dartboard. The next parameter is the median. This is where the 50th value would be if you put all of the results in order from lowest to highest, or alternatively half the darts will be to the left, half will be to the right. This represents the centre of the distribution – in this case right in the middle. Finally there are the lower and upper quartiles. These determine the shape of the distribution and are the values at which darts number 25 and 75 respectively are in our darts scenario, as they represent the values that 25% and 75% of the results are further left than.

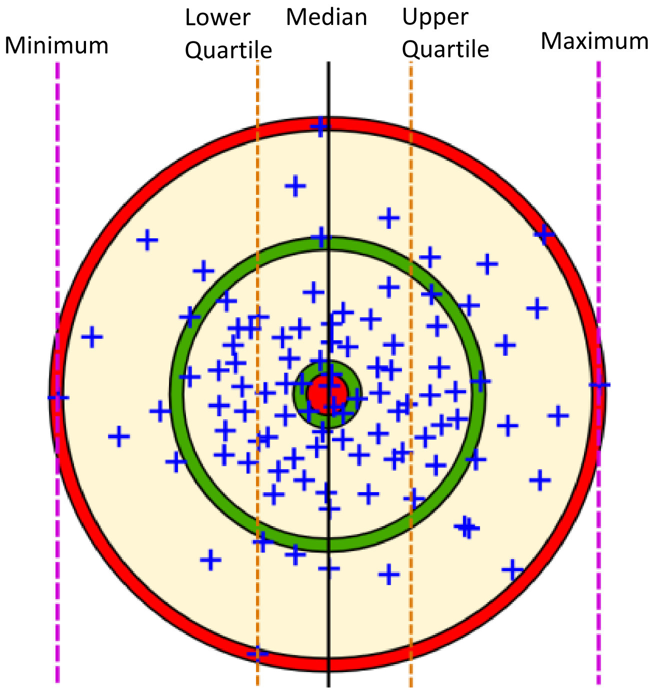


Figure 2: Distribution of dart throws with the key statistical parameters marked

This dartboard situation represents what is called a symmetrical distribution of results, as the darts are just as likely to hit on one side of the bullseye as the other. This is not always the case in geology.

Picture the darts scenario again, except in this case imagine that there is a steady crosswind from left to right. What would you expect the change in the distribution to be?

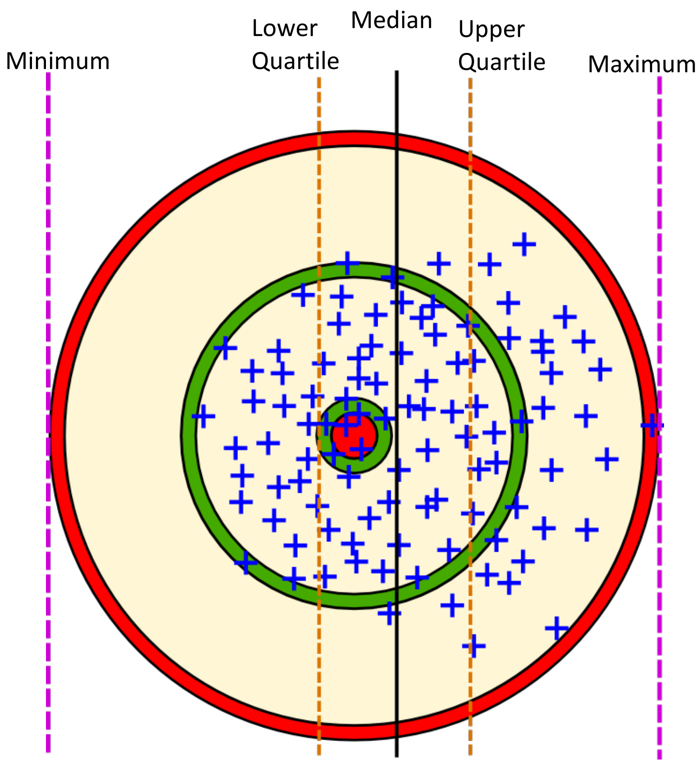
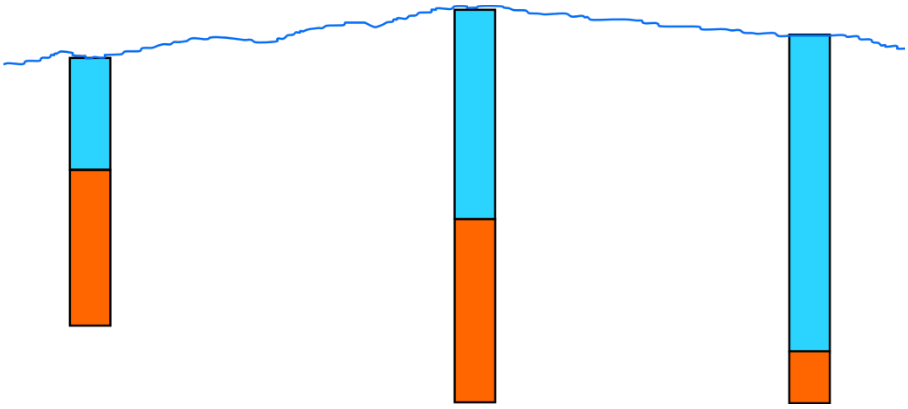


Figure 3: The same dartboard but with a bias introduced that creates a tendency for the darts to hit to the right of the target

In this case the wind has introduced a bias that has resulted in the distribution shifting to the right with ~65% of the darts hitting to the right of the bullseye. In our borehole situation this bias represents a tendency to over or under estimate the elevation of the surface.

Worked Example

Consider the previous case where we have an interpreted contact between two boreholes but with only the interpretation at the point of interest shown:



Recall that we are considering 100 independent notional instances of the same setting. At each of the 100 we go and drill a real borehole, and find the actual elevation of the contact of interest. In the Figure below we show the position of the contact from each of the 100 boreholes superimposed on the expert's interpretation. It is the distribution of these that we want to consider.

We start by considering the maximum and minimum – the highest and lowest possible elevation of the contact, relative to the interpretation.

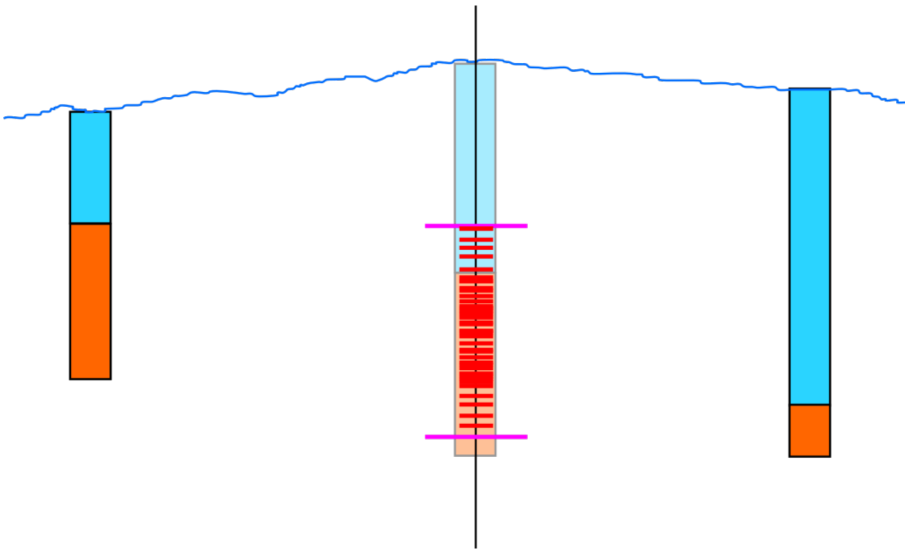


Figure 4: The example set of boreholes but with the elevations of the contact in our 100 notional boreholes marked in red and the maximum and minimum marked in pink

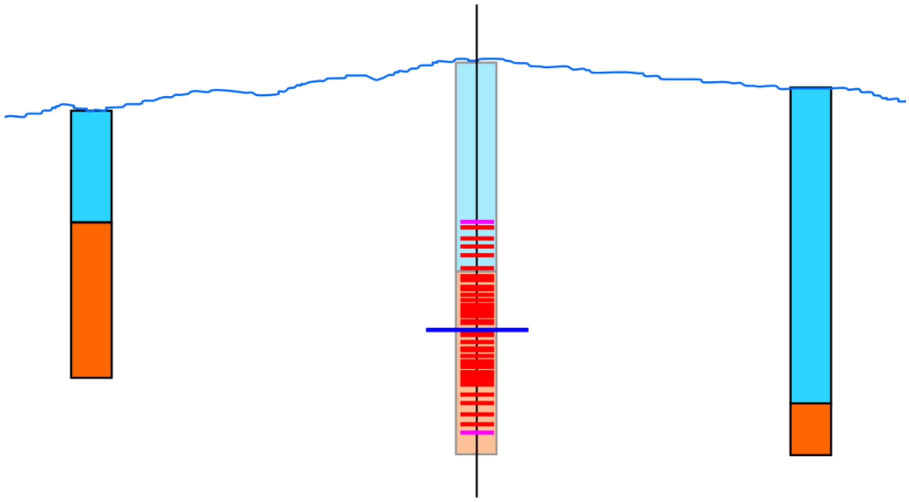


Figure 5: The interpreted boreholes with the median result marked in blue

Now we consider where the median elevation over the 100 instances. This is the elevation, s, such that in half the cases the contact is deeper than s and in half it is shallower.

Finally we consider where the upper and lower quartiles are. The lower quartile is at depth s_{q1} so that in a quarter (25) of the cases the contact is deeper than s_{q1} and in three quarters (75%) it is shallower. The upper quartile is at depth s_{q3} so that in three quarters (75) of the cases the contact is deeper than s_{q3} and in a quarter (25) it is shallower.

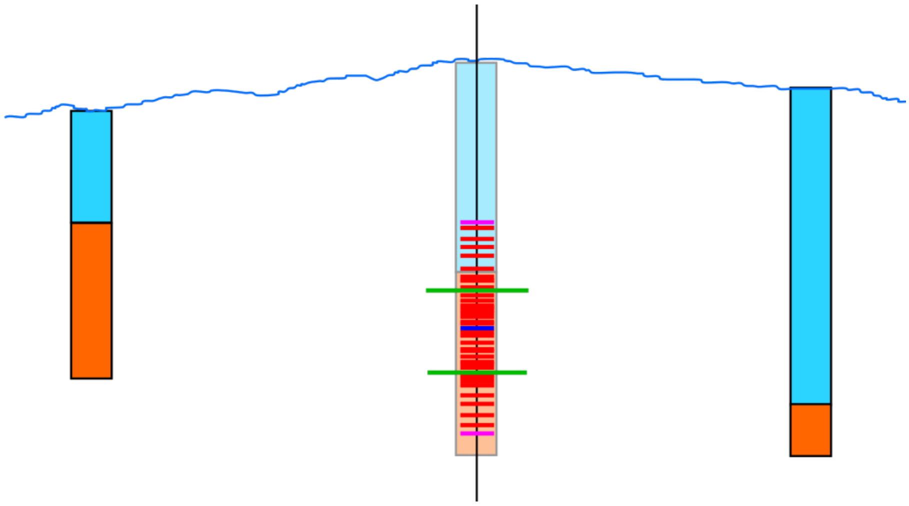


Figure 6: The interpreted boreholes with the upper and lower quartiles marked in green

Now we have those 5 pieces of information we can create a probability distribution curve, which indicates the probability of an interpretation being at a certain elevation. This curve can then be compared to the distribution curve of the error in the cross section experiment, allowing elicitations and cross section experiments to be compared.

How the elicitation will be run

We shall start with a briefing session in which you can raise and clarify any issues with this document or the scenarios. We will then carry out a practice elicitation of something non-geological to demonstrate how the process works.

The SHELF elicitation framework specifies a standard way in which elicitations are to be carried out and will be followed in this elicitation. For every scenario you will first be asked to discuss as a group what you think the maximum and minimum (highest and lowest values) would be.

You will then be asked to write down your own opinion on what the values for the median, lower quartile and upper quartile are. These will then be anonymously presented to the whole group for discussion, with the objective being a group consensus for each of those values. A final probability distribution will then be created.

Once this has been carried out for each scenario the probability distributions that you have created will be presented again for final confirmation of the results. Finally there is an opportunity to provide any feedback on how the experiment was run.