

Precision and Accuracy of Modal Analysis Methods for Clastic Deposits and Rocks: a Statistical and Numerical Modeling Approach

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Supplemental File 1

A REVIEW OF SOME PREVIOUS STUDIES AND STATISTICAL CONCEPTS

The Binomial Distribution to Statistically Model Point Counts

Point counts have often been studied using statistics, but we have not seen an equivalent treatment for line counts, and we do not attempt one here. In the paper, we only use statistical approaches to study point counts.

The binomial distribution describes a Bernoulli process, in which there are two possible outcomes for each trial: success or failure. For example, we randomly draw balls from a bag which contains blue and red balls, and drawing a blue ball is a ‘success’, with probability p , which is of course the proportion of blue balls. The selected ball is placed back in the bag and the balls are mixed after each draw, so each draw is independent. The number of trials, N , is fixed and known in advance, and the number of successes is discrete. If (i) a rock or deposit contains only two components, (ii) the point counting grid is larger than the largest component, and (iii) the rock or deposit is homogenous, then point counting is a Bernoulli sampling process (Neilson and Brockman, 1977; Weltje, 2002) and the binomial distribution applies¹.

The binomial distribution is a discrete function, unlike the normal distribution which is continuous. The binomial population mean is $\mu = Np$ and its population standard deviation is $\sigma_{nb} = \sqrt{Np(1-p)}$, in terms of the *number* of successes (Taylor, 1997; Howarth, 1998). For example, suppose that we count $N = 400$ points in a homogeneous thin section which contains blue and red particles and the true proportion of blue particles is $p = 0.5$. If we count these 400 points once, we will find x blue particles, somewhere close to 200, but probably not

¹ In some cases, the binomial distribution may not be a relevant statistical model for point counting, for example if the thin section, specimen or area under study is not internally homogeneous, or if the observations are not independent from each other, as would be the case if the same grain is counted twice (Weltje, 2002), or if the rock has some structure, like in a gneiss. However, the following discussion will assume that the binomial distribution applies. A generalized version of the binomial distribution, where more than two components are present, is the multinomial distribution (Howarth, 1998; Weltje, 2002).

exactly 200. If we repeat the count many times at different places on the thin section, we expect to count an average of $Np = 200$ blue particles if the method is accurate. According to statistical theory, the standard deviation of the measurements, s_{nb} , should be equal to that of the population (Weltje, 2002): $\sigma_{nb} = \sqrt{400 * 0.5 * 0.5} = 10$ blue particles.

If we instead express the population standard deviation as a *proportion* of blue particles, we divide it by N and the formula becomes $\sigma_{prop} = \sqrt{\frac{p(1-p)}{N}}$. In our example, σ_{prop} on the proportion of blue particles would be $\sqrt{\frac{0.5*0.5}{400}} = 0.025$, and the relative standard deviation would be $0.025/0.5 = 5\%$. Should we actually repeat the experiment a very large number of times, the measured standard deviation s_{prop} should equal σ_{prop} . We see that σ_{prop} , and therefore that the predicted counting error (assumed to be equivalent to precision in this line of reasoning), depends on N and p , with σ_{prop} decreasing with the square root of N . This is the basis of the “error chart” of van der Plas and Tobi (1965), which plots $2\sigma_{prop}$ values, absolute or relative (Fig. 2b in the paper). Continuing with our example, we use $N = 400$ and $p = 0.5$ on the chart and read absolute $2\sigma_{prop}$ halfway between the continuous lines for 4% (0.04) and 6% (0.06), i.e. about 5% (0.05). Or if we are interested in the relative value of $2\sigma_{prop}$, we find that the dashed line for 10% relative passes directly through our coordinates. Implicit in constructing this chart is the assumption that the binomial distribution can be approximated with a normal distribution so that the standard deviation can be used to build confidence intervals for a binomial distribution.

Confidence Intervals for a Binomial Proportion

A **confidence interval** is “an interval of plausible values that a population parameter might assume, based on the value of a statistic which estimates that parameter” (Davis, 2002). Here the population parameter of interest is the actual proportion p of a constituent and the statistic is the measured proportion $\hat{p} = x/N$ from componentry methods. The confidence interval is built so that there is a specified probability (confidence level), such as 90% or 95%, that the true value falls within the interval. The significance level (or desired level of risk), α , is defined as 1 minus the confidence level.

If the normal distribution is a good approximation of the binomial distribution, then confidence intervals for point counting and other componentry methods can be estimated using the standardized normal distribution (Vollset, 1993). For example, the $2\sigma_{prop}$ values in the van der Plas and Tobi (1965) chart would correspond to 95.45% confidence intervals. In other words, for a certain point count, there is a 95.45% chance that the true proportion of a component lies within $2\sigma_{prop}$ on each side of the counted proportion \hat{p} . In generalized form, the width of the confidence interval for a proportion is $Z \sigma_{prop}$ on either side of the measured proportion, where the Z -score corresponds to a specific cumulative probability for the standardized normal distribution (Vollset, 1993; Howarth, 1998). These scores can be found in tables within statistical textbooks (e.g. Davis, 2002). For example, for a 95.0% confidence level, $\alpha = 0.05$, but this is a two-sided interval, so we look up the Z -score for $1 - \alpha/2 = 0.9750$ cumulative probability and read 1.96. This means that the population value p is expected to lie within $1.96\sigma_{prop}$ on either side of \hat{p} , 19 times out of 20. The Z -score for a 68.27% confidence level is of course 1.00 and that for a 95.45% confidence level is 2.00. This traditional method of using the normal distribution to find confidence intervals for a binomial

proportion is called a “Wald interval” in the statistics literature (e.g., Vollset, 1993; Brown et al., 2002; Park and Leemis, 2019).

The normal approximation is very convenient, but is not always applicable, for example for very low or very high abundances, for very low number of points counted, or when high levels of confidence are sought (Vollset, 1993; Howarth, 1998; Weltje, 2002). One reason is that the normal distribution is symmetrical, but the binomial distribution is not, except if $p = 0.5$ (Taylor, 1997). A common criteria is that both Np and $N(1-p)$ must be larger than 5 or 10 for the normal approximation to apply (e.g., Howard, 1993; Vollset, 1993). However, the use of this rule is discouraged by Vollset (1993). [Figure S1a](#) shows two examples of Np smaller than 5, where the normal and binomial distribution diverge markedly, but even at $Np = 10$, the difference is noticeable. Brown et al. (2002) write that “the standard method in universal use [the normal approximation] is riddled with problems; so much so that it cannot be salvaged.”

With a binomial distribution, the confidence bounds are asymmetrical, i.e. the lower bounds and upper bounds are not equal (Howarth, 1998), except when $p = 0.5$. There are many different ways to obtain lower and upper confidence bounds for the binomial distribution, as discussed for example by Vollset (1993), Howarth (1998) or Park and Leemis (2019). We show three of them for a proportion of 0.1 and confidence levels of 68.27% and 95.45% in [figure S1b](#). For the Wald interval (normal approximation), the confidence bounds are symmetrical and equal to one or two standard deviations, respectively, for these confidence levels. The two non-normal binomial estimators shown are the Clopper-Pearson and the Wilson-score. In both cases, the confidence bounds are asymmetrical. The Clopper-Pearson method aims to provide coverage equal to or higher than the specified confidence level (Park and Leemis, 2019). In other words, it is a conservative method that may overestimate the width of the confidence bounds to remain “on the safe side”. Although this method was recommended by Howarth (1998), we do not use it further in this article. Finally, the Wilson-score method is recommended by Park and Leemis (2019) when N is 49 or more and the goal is to “minimize the absolute difference between the stated and actual coverage”, which is what we aim for here. At very large N , all of these methods converge.

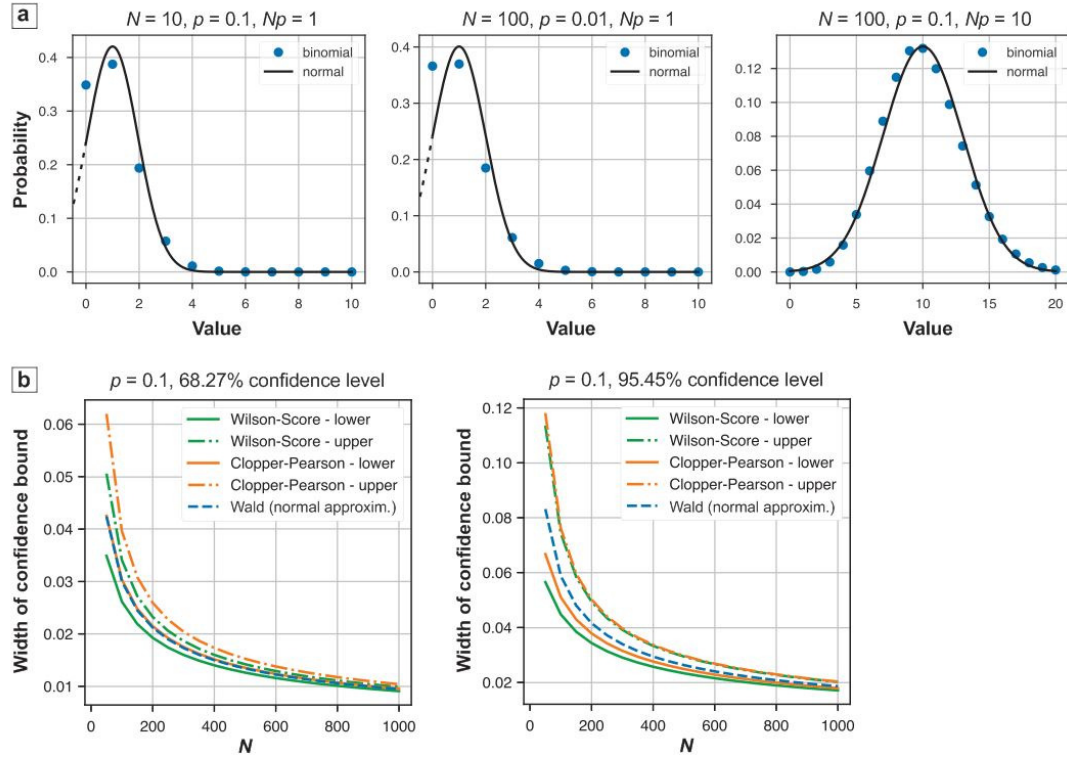


Figure S1. (a) Binomial distribution (blue dots) showing the probability of a certain number of successes, for different combinations of population proportions (p) and numbers of trials (N). A normal distribution with the same mean and standard deviation is shown for each plot (black bell curve). (b) Different ways of estimating confidence intervals for a binomial proportion. These last two graphs were drawn using a Python code translated from the `conf` package 1.7.0 for R of Christopher Weld and collaborators (<https://CRAN.R-project.org/package=conf>), more specifically the `binomTest` command, as used in Park and Leemis (2019).

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