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Supplementary Materials for
Toxic mercury pulses into the Late Permian Environment

Stephen E. Grasby*, Xiaojun Liu, Runsheng Yin, Richard Ernst, Zhuoheng Chen
Correspondence to: steve.grasby@canada.ca

Materials and Methods

1. Methods for Multi-scale fractal eruption model

The volcanic eruptions are generally assumed to be a stochastic process that distribution between sequential eruptions (de Silva et al., 2015; Halsey et al., 1986; Papale, 2018; Roy et al., 2012). Here we create a volcanic eruption model as the primary source of Mercury (Hg) emissions to atmosphere, and the observed scaling properties of temporal clustering in volcanic eruptions allow for the use of a multifractal approach to model the processes (Cheng, 2014; Gusev and Seismology, 2014; Kirichenko et al., 2019). A multifractal process $X(t)$ is an inhomogeneous fractal, the statistical moments of the probability density scale can be described as (Halsey et al., 1986; Kirichenko et al., 2019):

$$E[|X(t)|^q] \propto t^{q\tau(q)} \quad (S1)$$

where $E[\dots]$ is the random function independent of $X(t)$, $\tau(q)$ is the generalized scaling exponent, and t is the time series.

The binomial multiplicative cascade processes (Cheng, 2014; Kirichenko et al., 2019) is used to create the stochastic multifractal time series of volcanic eruptions. The initial unit time interval is divided into two equal sub-intervals. A weight coefficient, w_1 is assigned to one and $(1 - w_1)$ to the other. At the next iteration, two new random values w_2, w_3 are generated, for creating four sub-intervals with weights $w_1 w_2$, $w_1(1 - w_2)$, $(1 - w_1)w_3$, and $w_2(1 - w_3)$. The cascade time series with multifractal properties is generated after a number of iterations:

$$w = \left\{ \begin{array}{c} 1 \\ w_1, (1 - w_1) \\ w_1 w_2, w_1(1 - w_2), (1 - w_1)w_3, (1 - w_1)(1 - w_3) \\ \vdots \end{array} \right\} \quad (S2)$$

The weight w_i is a random variable from a Beta(α, β) distribution with the following form:

$$p(x) = \begin{cases} \frac{1}{\text{Beta}(\alpha, \beta)} (1-x)^{\beta-1}, & x \in [0,1] \\ 0, & x \notin [0,1] \end{cases} \quad (\text{S3})$$

Using different Beta parameters α and β to adjust the multifractal properties of time series (Fig. S1), we can calculate the density distribution and determine the scaling exponent $\tau(q)$ (Godano and Civetta, 1996; Kirichenko et al., 2019; Riedi, 2002):

$$\tau(q) = -\log_2 \frac{\text{Beta}(\alpha+q, \alpha)}{\text{Beta}(\alpha, \alpha)} - 1. \quad (\text{S4})$$

Fig. S2. shows a typical cascade time series based on the Beta distribution with parameters $\alpha = \beta = 3$. A LIP eruption model has two basic parameters, discharge function $I(t)$ and scaling exponent $\tau(q)$ (Godano and Civetta, 1996; Roy et al., 2012). We used a skew normal distribution $I(t)$ to generate the eruptions. The total mercury mass released during a major event and the associated subsequent smaller eruptions is fixed at 399,200 Gg (Grasby et al., 2015) as estimated from the Siberian trap and is distributed by the generated fractal model. Recent publications show the Siberian Trap LIP lasted about 300 ka, from -252.2 to -251.95 ma (Burgess and Bowring, 2015; Streets et al., 2011). For a given total amount of Hg released in a major LIP, we are interested in the relationship between eruption intensity and Hg release rate by varying the length of eruption last time of the LIP. By assuming five different eruption time lengths [40 ka, 80 ka, 160 ka, 320 ka, 640 ka], the proposed multi-fractal scale cascade model generated five time series of mercury releases corresponding to the different eruption lengths in Figure S3. A background geogenic emission, 90 Mg/a, is added to emission modelling when we run the cycle model (Amos et al., 2013; Streets et al., 2011).

2. Methods for global Hg cycle model

The seven-reservoir box model of the global mercury cycle is developed by focusing on individual components of the Hg cycle (Amos et al., 2013; Sun et al., 2019). Each mass transfer between modeled reservoirs is represented by a system of coupled first-order differential equations:

$$\frac{dm}{dt} = \mathbf{K}m + s \quad (\text{S5})$$

$\mathbf{K}$ is a 7x7 matrix of first-order time-invariant rate coefficients k_{ij} , m is reservoir masses of Hg, s is a vector describing the external forcing from the deep mineral reservoir. The rate coefficients of global mercury cycle is based on Figure 1 in reference (Amos et al., 2013) (Fig. S4). The natural eruption mass are used to derive the initial rate coefficients, (Table 2 in Amos et al., 2013). The programming language Python with open source libraries was chosen to implement the modeling.

Fig. S1. Density distribution and scaling exponent with different parameters.

Fig. S2. Generated multi-scale fractal distribution.

Fig. S3. Mercury emissions of Siberian LIP based on fractal distribution, different periods are assumed from 40 ka to 640 ka.

Fig. S4. The Hg flux coefficients driving the reservoir cycle model (based on Amos et al. 2013).

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